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A COMPANION TO THE AZIMUTH TABLES.

H. B. GOODWIN.

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A COMPANION TO THE AZIMUTH TABLES

BY

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THE VARIOUS AZIMUTH METHODS CONSIDERED WITH REFERENCE TO CONDITIONS FAVOURABLE TO EACH. FORMULÆ FOR EXTENDING THE USE OF THE ORDINARY TABLES TO LATITUDES AND DECLINATIONS HIGHER THAN THE TABULATED VALUES. NEW AND ORIGINAL METHODS FOR THE SOLUTION OF LEADING PROBLEMS IN NAUTICAL ASTRONOMY

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PREFACE.

The very general adoption of the Sumner methods for fixing position at sea in recent years has invested the problem of the Azimuth with a degree of importance much greater than that which was formerly attached to it, so that the space which is spared to the subject in the ordinary text books of the day seems hardly commensurate with the prominent position which it necessarily occupies in the methods of Navigation now generally practised. In the different Manuals various processes for the determination of Azimuth are placed before the student, but very little guidance is afforded as to the conditions suitable for the employment of one or other of these to the greatest advantage.

In the present little work an effort is made to deal with this deficiency, and to do as much for the Azimuth as has already been done for the hour angle, that is, to trace the effect on the Angle at the zenith of unavoidable errors in the data, from which deductions may be drawn as to the conditions favourable for the employment of one or other of the methods in a particular case.

Without special reference to the Azimuth in the final chapter the opportunity has been embraced of bringing together in a more permanent and collected form a number of more or less original notions in Nautical Astronomy that have been proposed during the last 35 years in the pages of the *Nautical Magazine*, the *Journal of the United*

States Naval Institute, and elsewhere. The late Admiral Sir Leopold Heath, in the historic letter introducing the renowned A.B. Tables, dated "H.M.S. *Iris*, Hong Kong, 27th June, 1845," wrote as follows:

"Many useful Rules and Tables are comparatively lost in the back numbers of the *Nautical Magazine*, and I think editors of future editions of Navigation books would do good service to the Naval world by availing themselves of some of them."

The *Nautical Magazine* was then in its first youth, the fourteenth year of its issue; it is now not far off its centenary. But the remark of Sir Leopold holds with equal force to-day.

In conclusion, the author wishes to tender his most sincere acknowledgments to Captain H. Lloyd Jenkins, R.N.R., F.R.A.S., a Principal Surveyor of Shipping under the Board of Trade, for his great kindness in reading through the proof sheets, and much valued assistance generally in the preparation of this little volume.

H. B. G.

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A Companion to the Azimuth Tables.

CHAPTER I.

Some Leading Features of the Azimuth Tables.

Within the last half century the problem of the Azimuth, which, at one time, occupied but a secondary position in Nautical Astronomy, has vastly increased in importance. Originally employed to find the Variation of Compass, the substitution in so many cases of iron ships for wooden ones led to the more frequent application to the determination of the troublesome and constantly changing element of Deviation; finally, the very general adoption of the "Sumner," or position-line methods, rendered a knowledge of the Azimuth of an observed body almost equally indispensable with the altitude itself, in each and every observation at sea.

Within living memory a navigating officer generally contented himself, to check behaviour of compass, with an observation for Amplitude, the distance of Sun from East or West point when on the horizon, and a Table of Amplitudes for latitudes 0° to 65°, declinations 0° to 23½° (Table 59) was given in the first edition of Raper in 1840. In addition to this Raper gave a method of deducing Azimuth from altitude by means of a "Spherical Traverse Table (No. 5)." This was intended for use in combination with the ordinary Plane Traverse Table, but the process was a somewhat

complicated one, giving almost, if not quite, as much trouble as the ordinary solution of the problem. Somewhere about 1860 the Admiralty appear to have commissioned the late Staff Commander Burdwood, in the course of his ordinary duties in the Hydrographic Department of the Admiralty, to prepare a set of Azimuth Tables for exact degrees of latitude 30° to 60°, and of declinations corresponding to the solar range of 0° to 23°. The hour angle Argument proceeded by intervals of four minutes, and values were not continued beyond an altitude of 60°. The "Advertisement" to the First Edition runs as follows:—

"To the practical navigator the ready determination of the Sun's true bearing, or azimuth, has always been deemed of great importance, but more especially will the advantage of tabular results be appreciated at the present time, when iron enters so largely into the construction of ships of war and merchant vessels, whereby the local magnetic influence on the compasses is greatly increased.

"In the *Azimuth Tables* by Thomas Lynn, Commander in the late East India Company's Service, which were published in 1829, the Sun's altitude is the chief element involved for finding the Azimuth, the Time not being employed."

Thus it would seem that the earliest form of Table had the altitude as argument, in place of hour angle.

Captain Burdwood appears to have carried out his onerous task with wonderful fidelity, working out each result with six figure logarithms to the nearest minute of arc. The result of these labours was issued in sections as completed, and it was not until 1866 that the great work was published in a complete form.

Nine years elapsed before the publication of the companion volume, which covers the zone of latitude extending from 30° N. to 30° S. arranged precisely on the same plan as the original Burdwood. These Tables were computed jointly by Captain John E. Davis, a colleague of Burdwood at Whitehall, and his son Mr. Percy H. L. Davis, who until recently was Chief Assistant in the *Nautical Almanac* Office.

In the Preface, dated "Greenwich 1875" the following passage occurs:—

"It is but right to state that it was the intention of the late Staff Commander Burdwood to have completed the work he had so ably commenced, and at his death he left a portion of the Tables computed, but it was unknown to the authors until this work was in the press."

The publication of the Davis Tables brings us to the adoption of the Azimuth Tables for the purpose of laying off position-lines, which is beyond all doubt an important landmark in the history of Modern Navigation. In the original Preface to the Davis Tables there is a memorandum by Staff Commander J. C. Richards, R.N., as follows:

"Briefly stated, Sumner's method involves finding—by means of an observed altitude and *two* assumed latitudes—two longitudes, the line on the chart so obtained being called the *line of position*. A second observed altitude, separated by a suitable difference of bearing, gives by similar treatment another line of position, the intersection of which with the first line of position, *projected for the run of the ship in the interval*, determines the place of the ship at the second observation. The line of position can however be as readily obtained from *one* assumed latitude, if the true bearing of the Sun, or other celestial object, be known;

for the direction of the line of position differs by 90° from the direction of bearing of the object."

To which in a footnote this further piece of information is appended:

"Captain Sumner remarked in his 'New Method of Finding a Ship's Position at Sea, p. 29.' If the Sun's azimuth could always be exactly observed, it would be necessary to find the longitude of only one point in each straight line . . . . but except under very favourable circumstances this cannot be done with sufficient accuracy, and the only safe way is to find the position of the two points in each line as directed, *i.e.*, by working each observed altitude with two assumed latitudes."

"The practical value of this application of the Azimuth Tables to Sumner's method," continued Captain Richards, "is that the computation is shortened one half." It may well be doubted indeed whether the calculation of four hour angles, as required in what is known as the "Double Chronometer," is not fully as long and tedious as would be the calculation of a Double Altitude of the old style, combined with the longitude furnished by the greater bearing observation, which together give the most accurate position possible supplied by a pair of observations at sea.

The Tables "For the Higher Declinations,"* compiled by the present writer, made their appearance in 1896, rather more than 20 years later than the Davis Tables. It is a feature of these Tables that for the larger bearings in Table A the altitude is employed as Argument, but in Table B, for observations near the meridian, the Hour Angle. The object

* *Azimuth Tables for Higher Declinations.* (Limits of Latitude 0° to 60°, of Declination 24° to 30°.) James Brown & Son (Glasgow) Ltd. Price 7s. 6d.

for this distinction in point of Argument is thus stated in the Preface: "As has been already stated Table A is not continued after the altitude has come within 2° of the meridian altitude. The reason for this will be readily understood, for, as the body approaches the meridian, its motion in altitude becomes so slow in comparison with the change of azimuth that the altitude can no longer be regarded as a suitable Argument for the Table.

To meet this difficulty Table B has been computed. Commencing at one hour from the meridian, it gives the Azimuth corresponding to each four minutes of hour angle for the various degrees of declination (24°—30°) of Contrary Name to the latitude of the place. Table B is limited to the case of Contrary Name, for the reason that near Meridian the altitudes of bodies with declination from 23° to 30° of Same Name are too high to enable the Tables to be employed with advantage. Later on a new and original method will be described, which enables the Azimuth for Same Name to be deduced from that recorded for the given latitude and declination of Contrary Name in an exceedingly simple manner.

The following stars of first and second magnitudes fall within the limits of declination dealt with in the Tables.

Star	Magnitude Jan. 1924.	R.A.	Dec.
α Andromedae	2.2	0h 4m	28° 40′ N
β Tauri	1.8	5 22	28 33 N
ε Canis Majoris	1.6	6 56	28 52 S
δ Canis Minoris	2.0	7 5	26 16 S
β Geminorum (Pollux)	1.2	7 41	28 12 N
α Coronae Borealis	2.3	15 31	26 58 N
α Scorpii (*Antares*)	1.2	16 25	26 16 S
α Piscis Australis (*Fomalhaut*)	1.3	22 53	30 2 S
β Pegasi (*Scheat*)	(2.2—2.7)	23 0	27 40 N

The following note from the "Introduction" with regard to this list of stars may be of interest:—

"In connection with these stars of Southern declination, the Tables may be expected to be particularly useful for two reasons; firstly, because we have no Pole Star in the southern hemisphere, to furnish a ready means of obtaining latitude at any hour of the night; and secondly, because that zone of the heavens which extends from the Equator to 23° S. is not well furnished with bright stars. Burdwood, in his Introduction, mentions only three such stars as between declinations 0° and 23° S., while between the parallels of 23° and 31° we have as many as four, including the two brilliant stars Antares and Fomalhaut."

With regard to the selection of the altitude as Argument in Table A it may be remarked that such Tables provide the simplest possible method of finding deviation, the somewhat troublesome business of finding the hour angle of the body being wholly eliminated. As Captain Lecky observed in the *Wrinkles*:

"It is perfectly wonderful how few men avail themselves of the stars on a fine night, to see how their compasses are behaving. This arises principally from an ill-defined idea that any problem connected with the stars is too difficult to be meddled with. How different are the actual facts!"

Amongst other uses of the Tables for High Declinations they may at certain periods be most advantageously employed in connection with our satellite the Moon. Next to the Sun the Moon is gradually coming to be recognised as the individual heavenly body of greatest importance at sea, for the simple reason that observed with the Sun in the daylight she offers the only means of obtaining a complete "fix" at

once, without waiting for the change in bearing which is essential when two observations of the same body are involved. Now the Moon's range in declination is liable to considerable variation. The limits are sometimes as low as 18° N. and S., at others they extend to 28° in each direction. Perhaps a few words may be introduced here upon the nature and cause of these variations, as it is a subject hardly ever touched upon in elementary works on general Astronomy.

The plane of the Moon's orbit is inclined at an angle of a little over 5° to the ecliptic, and the value of maximum declination for a particular month depends upon the position of a point called the "Node," or point of intersection of the Moon's orbit with the Ecliptic. There will be two of these Nodes, the ascending one and descending one for each month. The value of the greatest declination for the month is fixed by the position of the ascending Node as the Moon passes from South declination to North. The maximum declination possible is attained when the Node has longitude 0°, that is, coincides with the First Point of Aries, its value being then 23° 27′+5° 9′, or 28° 36′. Similarly the minimum occurs when the Node is in Libra, when the maximum value is given by the difference of the two quantities, or a little over 18°. These facts lend some interest to a column of figures given in the larger official *Nautical Almanac*, which otherwise might fail to engage our attention. The column is on page 1, and is headed "Mean Longitude of Ascending Node." Thus in the Almanac for 1915 we have the value given for January 1st 329°, with the Moon attaining a maximum declination of about 28° N. or S. As the Node is carried back about 19° 6′ year by year it would seem that it was situated near the First Point of Aries about the middle of

1913, and for several years before and after that year the maximum declination exceeded the solar limit of 23° for a more or less considerable portion of each month. At present the value given for the Node's longitude for 1st January, 1924, is 155°, only 25° from the position of the minimum in Libra, so that diminishing at the rate of 19·1° per year the First Point of Aries should be reached about 1931, but of course the maximum declination will shew an excess over the solar limit of 23°, month by month, several years before the Node reaches longitude 360°. For some ten years or so from the end of 1926, in fact, the Moon may be expected to exceed the solar limits of declination for a greater or less period during each month. Thus we are enabled to trace some connection between the Azimuth Tables and an apparently uninteresting column of figures disinterred from an odd corner of the Official Almanac, which at first sight might appear likely to be of little service to any one.

CHAPTER II.

Various Methods for Calculation of Azimuth Formulæ on Errors in Azimuth resulting from Errors in data. Deductions as to the Advantages of each under particular Conditions, and in Compilation of Azimuth Tables.

The student of Nautical Astronomy may well feel a little bewildered by the variety of methods presented to his notice for finding the Azimuth of a heavenly body. For the present purpose, however, attention may be limited to the methods described in Sub-head VII of Chapter IV of the Nautical Astronomy Section of Raper's *Practice of Navigation* (pp. 240, 241).

The methods in question are four in number:

(1) To find the Azimuth, the Altitude being given.

(2) To find the Azimuth, the Hour Angle being given.

(3) To find the Azimuth, the Hour Angle and Altitude being given.

(4) To find the Azimuth, not far from the Meridian by the observed change of Altitude in a small Interval of Time. Of these four processes the last mentioned may be at once dismissed from consideration. All that is said of it in Raper is:

"This method will sometimes be useful, as for determining the variation, but it must be employed with caution; the

interval should not be very small, the body should not be far from the meridian, and both altitudes must of course be observed on the same side."

The method has been handed down to us from a period when the hour angle at sea was often a very uncertain quantity and may be regarded as having passed off the stage. For the method which stands next above it in order when Azimuth Tables do not happen to be available there is a good deal to be said.

In considering the different processes it will be convenient to have before us a diagram of the Triangle of Position, which is given below.

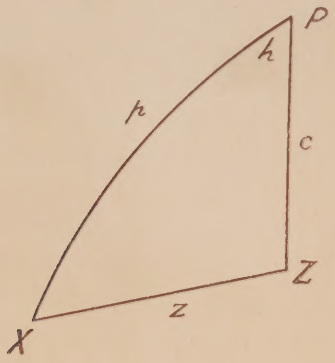

Fig. 1.

In the diagram P is the North Pole, Z the zenith, X the position of a heavenly body West of Meridian. In accordance with the usual notation $P\,Z$, or c, is the colatitude, $P\,X$, or p, the Polar distance of body, $Z\,X$ or z, the zenith distance, $Z\,P\,X$ is h the hour angle, and $P\,Z\,X$ or Z the Azimuth with which we are concerned. The angle at the body

$P\,X\,Z$ which will occur in our investigations is sometimes called the "Angle of Position."

Reverting to the case of the Azimuth as found by a combination of altitude and hour angle, we obtain from our triangle the formula—

$$\frac{\text{Sin } P\,Z\,X}{\text{Sin } Z\,P\,X} = \frac{\text{Sin } P\,X}{\text{Sin } Z\,X}$$

$$\text{or Sin } Z = \frac{\sin p}{\sin z} \sin h$$

Applying this formula to the practical example given in Raper for the data h=1h 19m 19s, alt=58° 40′, p=104° 24′ we have—

L Sine 104° 24′	9.9861
L Sine 1h 19m 19s	9.5305
Log. Sec. 58° 40′	.2840
Z=39° 11′ L Sine	9.8006

Those who in the absence of Azimuth Tables are in the habit of working out the bearing by the relatively laborious processes of either Altitude or Time Azimuth may feel something of surprise that a method so short and simple does not find universal acceptance, and it may be well at this stage to offer some explanation of the methods of investigation by which the conditions favourable or unfavourable to the employment of a particular problem are generally determined. It is upon the formulæ which result from enquiries of this nature that the "Degree of Dependence" commonly attached to the various problems in the "Practice of Navigation," and which has always been regarded as one of the most admirable features of that renowned treatise, are in the first instance based.

The Triangle of Position, like every other Spherical Triangle, has six parts. Any four of these being selected a formula may be obtained which connects the parts in question. In works on Spherical Trigonometry* fifteen such formulæ, under four separate types, are demonstrated, and as fifteen is the number of combinations that can be formed of six things taken four together it is obvious that every possible case is provided for.

The appropriate formula having been selected, two of the terms may be regarded as constant, that is, not liable to change. It is obvious that we cannot consider more than two as constant, since in that case the triangle would be completely determined. Two of the elements being thus fixed, by the ordinary process of differentiation we can obtain an expression, in some cases simple, in others perhaps somewhat tangled, which will furnish approximately the change in one of the elements, consequent on a small increase or decrease in another. It will often happen that a simple geometrical construction may be substituted for the process of differentiation, thus rendering the procedure adopted intelligible to a larger body of students. The investigations for error in hour angle consequent on errors in altitude and of latitude, given in the ordinary text books, are examples of this method of treatment.

The Azimuth by Altitude and Hour Angle combined.

Here we have the Sine formula, as already employed.

$$\text{Sin } Z = \frac{\sin p}{\sin z} \sin h$$

Now in the selection of the two constants we may at once

* See *Plane and Spherical Trigonometry.* By H. B. Goodwin. Longmans, Green & Co.

pick out p as practically correct, for there is not much doubt as to the accuracy of the declination value. Of the two others we are hardly likely to have the proper value for z coinciding with that for h, and we may assume either one or other to be the variable.

By differentiation we obtain, if $d\,Z$ is the change in Azimuth corresponding to an error $d\,h$ in hour angle,

$$d\,Z = \sin p \operatorname{cosec} z \cos h \sec Z\, d\,h$$

Two terms in this expression seem to invite attention, viz., cosec z and sec Z.

Now cosec z, it will be seen later, enters into each of the formulæ obtained for error, and as z approaches the value zero the cosecant becomes infinite. Any one who has attempted to take a meridian altitude with the Sun in the neighbourhood of the zenith will readily understand how hopeless it is to get an accurate bearing of a body which flies through several points in a minute or so. At any reasonable altitude, however, we need not trouble much about cosec z.

But sec Z is a different matter. Just as the cosecant becomes infinite at zero, so the secant is infinite at 90°. So that the hour angle and altitude method, excellent in general near meridian, loses its charm as the Prime Vertical is approached. But on small bearings it would seem that the method is deserving of more attention than it has received; in fact in many of the text books it is ignored altogether.

The Time Azimuth.

In the Time Azimuth we have a problem which can be dealt with by a geometrical construction in a very simple manner.

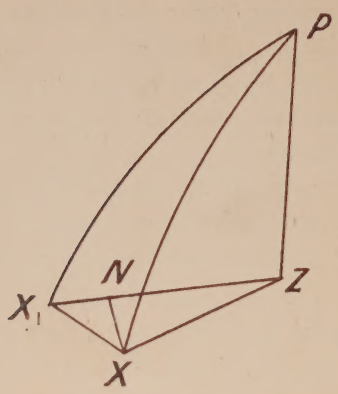

Fig. 2.

Let XX_1 represent a small portion of the diurnal path so that $P\,X_1 = P\,X$ and $P\,X_1X$ is a right angle. From X let fall a perpendicular arc $X\,N$ on $Z\,X_1$.

Then the angle $X\,Z\,X_1$ represents the change of Azimuth $d\,Z$ corresponding to a small change of hour angle $X\,P\,X_1$ or $d\,h$.

And

$$\begin{aligned} \text{Arc } X\,N = d\,Z \sin z &= X\,X_1 \sin N\,X_1X \\ &= d\,h \sin p \sin N\,X_1X \\ &= d\,h \sin p \cos P\,X_1Z \end{aligned}$$

Hence

$$d\,Z = \sin p \operatorname{cosec} z \cos.\ P\,X_1Z\ d\,h$$

Here we have a formula which gives a minimum of trouble. If z approaches zero indeed cosec z becomes infinite, and the method, in common with the other methods, fails altogether. With regard to the angle $P\,X\,Z$, when latitude is less than declination of body and of Same Name, $P\,X\,Z$ at its maximum becomes 90°, and since cos 90° = 0 $d\,Z$ vanishes. When the body is on the Prime Vertical $P\,X\,Z$ has its maximum value

for the day, and cos $P\,X\,Z$ is diminished accordingly. Near the meridian cos $P\,X\,Z$ approaches the value unity, but cannot exceed it. Generally it may be said that the Time Azimuth supplies a good all round, generally available, formula, failing only when, through the approach of the body to zenith, the problem of the Azimuth may be said to have no exact solution.

The Altitude Azimuth.

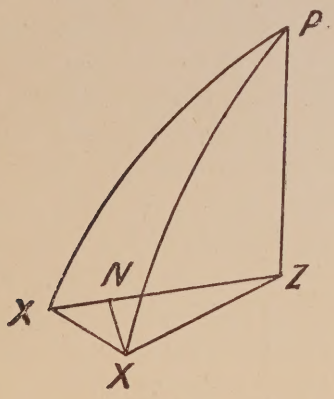

Fig. 3.

The case of the Altitude Azimuth may be dealt with by means of the diagram employed for the Time Azimuth. The angles $Z\,X\,N$, $Z\,N\,X$ being right angles $Z\,N = Z\,X$, and $N\,X_1$ represents the small change of zenith distance $d\,z$, the angle $X\,Z\,X_1$ representing the change of Azimuth as before.

Then

$$N\,X = d\,Z \sin z = N\,X_1 \tan N\,X_1\,X = d\,z \cot P\,X_1 Z.$$

Whence

$$d\,Z = d\,z \cot P\,X_1\,Z \operatorname{cosec} z.$$

The presence of cosec z as a factor, as in other methods, shews that the process is not available for altitudes approaching 90°.

With regard to the angle $P\ X\ Z$, as the body nears the meridian this Angle of Position continually decreases, and the cotangent tends to become infinitely great. The Altitude Azimuth therefore ceases to give satisfactory results for some time before the Time Azimuth becomes unreliable.

Near the Prime Vertical, however, when, as already mentioned, the Angle of Position has its maximum value, and on reasonably large bearings, there is little to choose between the two methods of Time and Altitude Azimuth, so far as accuracy is concerned. The most favourable condition in the case of both problems is presented by a body of Same Name, and of higher declination than the latitude of place, when at its maximum Azimuth. Here, as already remarked, the Angle of Position is a right angle, and neither an error in the hour angle, in the Time Azimuth, or an error in altitude in the Altitude Azimuth, produces any appreciable change in the Azimuth deduced.

The Amplitude.

The Amplitude may be regarded as a particular case of the Altitude Azimuth at a zenith distance of 90°. By reason of the small amount of trouble involved this method is universally popular, and Tables giving the value by inspection have certainly been included in Nautical Tables for at least a century. In the case of the Amplitude the conditions are bound to be favourable, for the pestilential cosec z has then its minimum value of unity. As for the

Angle of Position, in the case of Latitude and Declination of Contrary Name a body in the horizon has the maximum value for the day, while for Same Name in all navigable latitudes it is bound to have generally a reasonably large value.

Deductions as to Arguments for Azimuth Tables.

From what has been said certain conclusions may be deduced as to the Arguments best adapted to the compilation of Azimuth Tables. In order to avoid the inconvenience of the Double Entry, since the fundamental formulæ connecting the various groups into which the six parts may be distributed each contain four members, the Tables are necessarily somewhat bulky, but the convenience which enables us to obtain by simple inspection the result of what would otherwise be reached only by long and laborious calculation may perhaps be held to atone for this defect.

Of the three Arguments required to give us at sight what may be regarded as the "x" of our fundamental equation, two, the latitude of place and declination of body, may be considered indispensable. We have then only to discriminate between the claims of hour angle and altitude for adoption as the third. Of these for general purposes the hour angle is undoubtedly entitled to the first place. Other methods, as we have seen, have their weak points, failing near Prime Vertical and Meridian, as the case may be. But the Time Azimuth is always good alike, until the presence in our formulæ for error of the fatal "Cosec z," soaring into the regions of infinity at altitudes approaching 90°, warns us that the Azimuth is changing with such rapidity as to prevent us from fixing its value with certainty at any given moment,

whatever be the method employed. Most of the Tables in use to-day therefore are properly based on the hour angle. Tables like the original Burdwood, for each degree of latitude and declination, and for each four minutes of Time, equivalent to a degree of Arc, may be regarded as approaching such perfection as is possible in human affairs. The ten minute interval of Time, which has been adopted in some Tables of later date, permits an extension, within an equal number of pages, of the range of declinations treated, but introduces an element of interpolation, which is naturally to be avoided if possible.

Summing up the results of our investigations, these appear to lead to conclusions somewhat as follows:—

The Altitude Azimuth, with altitude, latitude, and declination as data (Raper, Art. 673) has this advantage that it is solved by the same formula of three sides given to find an angle with which every navigator is familiar in connection with the hour angle. For purposes of compass correction also a Table which has the altitude as argument offers the simplest possible method of finding the bearing of a star or other heavenly body at sea. But it is a somewhat serious drawback that the method becomes less and less reliable as the meridian is approached.

The Sine method by altitude combined with hour angle (Raper, Art. 676) is so beautifully simple that many, on making its first acquaintance, will be inclined to ask, "Why did I not know of this before?" Particularly in position-line work, since in that case both altitude and hour angle are ready to hand. But just as the Altitude Azimuth fails near meridian so does this Sine method begin to give trouble about the Prime Vertical, which, so far as longitude observations

are concerned, restricts its utility in no small degree. The Time Azimuth (Raper, Art. 675) is good alike near horizon, Prime Vertical, and Meridian alike, so that the Hour Angle undoubtedly offers the more convenient argument for a general Table, in place of the altitude which was formerly employed. The formulæ required for the logarithmic solution are perhaps somewhat troublesome, but various short methods, to some of which reference will be made later, have been put forward from time to time, and these greatly simplify the calculations required for declinations beyond the tabular limits. Without much hesitation therefore it is to the Time Azimuth, so generally available, and so thoroughly dependable, that the premier position amongst the various methods may fairly be assigned.

CHAPTER III.

Certain Short Methods whereby the Use of the Azimuth Tables is extended to cases outside the range of the ordinary Tables for Sun Limits of Declination, and Latitudes of 60°.

Owing largely to the energy and enterprise of a single individual, Mr. Percy L. H. Davis, formerly Chief Assistant in the *Nautical Almanac* Office, navigators are now provided with a fairly complete series of Azimuth Tables, ranging in respect both of latitude and declination from 0° to 78°. A complete outfit of these Tables, however, would be somewhat expensive, running to a total of several pounds sterling, and would form in themselves quite a little library, more or less burdensome for the ordinary navigator to carry with him about the world. It may therefore be considered a useful service to describe certain short and simple processes, whereby the sphere of operations of the ordinary Tables for moderate limits of declination and latitude may be extended to cases not contemplated in the first instance by the compilers of various Tables in common use. Prominent amongst these is the following problem, the special application of which will be pointed out later.

Having given the Azimuth of a body from the Tables for given values of Latitude and Declination of Contrary Name, to deduce the Azimuth for the same values of Latitude and

Declination of Same Name, the hour angle also being the same in each case.

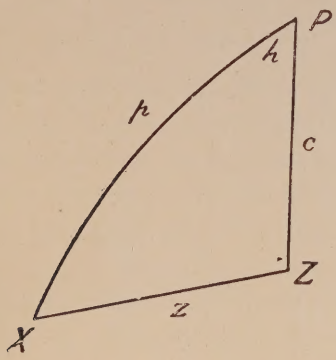

Fig. 4.

Let Z_1 represent the Azimuth at hour angle h of a body of declination d, of *Contrary Name*, so that p_1, the polar distance $=90° + d$. It is required to find the Azimuth Z_2, when the given values of latitude and declination are of *Same Name*, the hour angle being the same in each case.

Let p_2 denote the polar distance for *Same Name* so that $p_2 = 90° - d$.

From the typical "Triangle of Position" $P\,X\,Z$ we have

$$\text{Cot } p_1 \sin c = \cot Z_1 \sin h + \cos h \cos c$$
$$\text{Cot } p_2 \sin c = \cot Z_2 \sin h + \cos h \cos c$$

whence, adding the equations, and noting that

$$\cot p_2 = -\cot p_1$$
$$(\cot Z_1 + \cot Z_2) \sin h + 2 \cos h \cos c = o$$

and
$$(\cot Z_1 + \cot Z_2) = -2 \cot h \cos c,$$

or
$$\cot Z_2 = -\cot Z_1 - 2 \cot h \cos c.$$

From this equation, since Z_1, h, and c are all known we may find Z_2 as follows:

$$\text{Nat cot } Z_2 = \text{Nat cot } (180° - Z_1) - C$$

$$\text{where } C = 2 \cot h \cos c.$$

With regard to algebraic signs Nat cot $(180° - Z_1)$ will always be a positive quantity, because for *Contrary Name* Z_1 must always be greater than 90°, and therefore $(180° - Z_1)$ less than 90°. Also, if we limit the hour angle to values less than six hours, which will suffice for the special object in view, C will always be positive.

The process will therefore amount to taking the difference between two positive quantities and the angle Z_2 will in consequence be less or greater than 90° according to the numerical value of the terms on the right hand of the equation.

The special reason from which this process derives its importance arises from the fact that in the Tables for the Higher Declinations, both of Davis and Goodwin, within an hour of the Meridian, values for Azimuth are tabulated for *Contrary Name*, but not for *Same Name*. In the latter case the body would very generally be so near the Meridian that the tabular method for Azimuth would be unsuitable. It is therefore convenient to be able to readily deduce an Azimuth for *Same Name* from that given in the Tables for *Contrary Name*. To still further facilitate the deduction of one Azimuth from the other, the values of the quantity C have been tabulated for each four minutes of hour angle from 0h 8m to 1h 0m, and for each degree of latitude from 1° to 60°, and values for hour angle 0h 40m are given in the Table appended.

TABLE C.

The Factor which converts Azimuths for Contrary Name to Same Name.

Latitude	Hour Angle 0h 40m	Latitude	Hour Angle 0h 40m
1°	·198	31°	5·842
2	·396	32	6·011
3	·594	33	6·178
4	·791	34	6·343
5	·989	35	6·506
6	1·186	36	6·667
7	1·382	37	6·826
8	1·579	38	6·983
9	1·774	39	7·139
10	1·970	40	7·291
11	2·165	41	7·441
12	2·358	42	7·590
13	2·551	43	7·736
14	2·744	44	7·879
15	2·936	45	8·020
16	3·126	46	8·159
17	3·316	47	8·295
18	3·505	48	8·429
19	3·693	49	8·560
20	3·879	50	8·689
21	4·065	51	8·815
22	4·249	52	8·938
23	4·432	53	9·059
24	4·613	54	9·176
25	4·794	55	9·291
26	4·972	56	9·403
27	5·149	57	9·513
28	5·325	58	9·619
29	5·499	59	9·722
30	5·671	60	9·823

The example which follows will illustrate the method of procedure.

In latitude 44° N, when the declination was 28° N. and hour angle 0h 40m, required the Azimuth.

From Goodwin's *Azimuth Tables for the Higher Declinations* the Azimuth for Lat. 44° N., Dec. 28° S., is 170° 45′.

Then

From Table, for lat. 44°, hour angle 0h 40m C=7.879−
Nat cot 9° 15′ (180°−170° 45′) =6.140+

1.739−
Nat cot 29° 54′

Whence Azimuth required is 180°−29°54′ or N 150° 6′W.

Having given the hour angle and declination of a circumpolar star, of declination greater than 23½°, to find its Azimuth by means of the ordinary Tables.

Let p_1 be the polar distance (supposed North) of the star, h its hour angle, and c the colatitude of the place.

Let us suppose that in the same latitude, at the same hour angle, we can find the polar distance of a star which has for its Azimuth $180°-Z$, the supplement of the Azimuth required, then, if the declination of this fictitious, or imaginary star, falls between the limits of 23° N. and 23° S., we can find its Azimuth from the ordinary Solar Tables, and, subtracting the value so obtained from 180° we shall have the Azimuth which we seek.

Employing as before the fundamental formula connecting four adjacent parts of the spherical triangle we have the equations:

$$\cot p_1 \sin c = \cot Z \sin h + \cos h \cos c$$
$$\cot p_2 \sin c = \cot (180° - Z) \sin h + \cos h \cos c$$

p_1 being the polar distance of the real star, and p_2 that of the imaginary one.

Adding the two equations

$$(\cot p_1 + \cot p_2) \sin c = 2 \cos h \cos c$$

and

$$\begin{array}{rcl} \cot p_2 = & -\cot p_1 + 2 \cos h \cot c \\ & + \qquad + \\ & = -\cot p_1 + \mathrm{D} \text{ (suppose).} \end{array}$$

Since the star is of *Same Name* as the latitude with respect to declination, cot p_1 is always positive, and if, as before, attention is limited to the case of hour angle less than six hours, the quantity D is also positive, so that as in the preceding problem we have simply to deal with the difference of two quantities by means of the Table of natural tangents and cotangents.

Again since the quantity styled "D" in the present case is of the same form as the "C" of the previous problem, the value of D might be obtained by an extension of the Table already completed to the limit of one hour to a six hours' limit. Such an extension of the Table, however, seems hardly justified by the importance of the problem, particularly as only two logarithms are involved in the calculation of D.

Practical Example.

In latitude 32° N. it is required to find the approximate Azimuth of η Ursæ Majoris (*Benetnasch*), Dec. 49° 42′ N. at an hour angle of 2h 20m.

By the formula obtained

$\cot p_2 = D - \cot p_1$ where $D = 2 \cos h \cot c$.

To find the Value of D.

Log. 2	.301
L cos h	9.913
L cot c	9.796
(Sum)	20.010
Nat No. (D)	1.024+
Cot p_1	1.179−
(Diff) cot p_2	.155−
p_2	98° 49′

whence the declination=8° 49′ S.

Entering Burdwood with Lat. 32° N., Dec. 9° S., Hour Angle 2h 20m, we have for Azimuth 134° 44′, which, subtracted from 180°, gives N. 45° 16′ W. as the value required.

A short method for finding D by means of the Towson Tables.

The calculation of "D" may be materially simplified by means of that wholly admirable, but insufficiently appreciated, work, the Towson Great Circle Tables*. These Tables present a complete solution of the right-angled Spherical Triangle, and may be employed, as was shewn in the pages of the *Nautical Magazine* some years since, for a great variety of purposes unconnected with the use in Great Circle Sailing, to promote which was the primary object of their calculation.

In these Tables each Great Circle of the Sphere is identified by its Vertex, the point at which the circle most nearly approaches the Pole, when the Course on the circle is due East or West. In the diagram given below V is the vertex of the circle, the point at which the circle most nearly

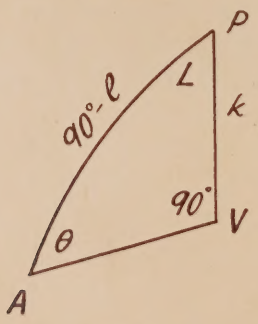

Fig. 5.

Tables to Facilitate the Practice of Great Circle Sailing. By John Thomas Towson. J. D. Potter, 145 Minories. Price One Shilling.

approaches P the Pole, and k the colatitude of the Vertex, which for a particular column is regarded as constant. Values are given in the Table for l, the latitude at a given point A, for θ the course at the point, and for $A\ V$ or d, the distance in nautical miles from the Vertex of the circle. The third Argument of the Table in addition to k, the colatitude of Vertex, and the right angle at V, is L, the angle $A\ P\ V$, representing the difference in longitude between A and the Vertex. For successive values of L, proceeding by single degrees, values for latitude, Course, and Distance from Vertex are recorded in the Tables.

To find an approximate value for D.

From the formula given above

$$D = 2 \cos h \cot c$$

$$\text{Let } \cos h \cot c = \tan \beta \quad (1)$$

Then from the diagram

$$\text{Cos } L = \tan k \tan l_1, \text{ if } l_1 \text{ is the latitude of } A$$

and

$$\cos L \cot k = \tan l_1 \quad (2)$$

Comparing the equations (1) and (2) we see that if in the Tables we take the latitude of place of observation for latitude of Vertex, and hour angle h as the Diff. Long. L, the tangent of latitude at the point A, will be the value of tan β required.

Reverting to the example worked above we have for the data Lat. 32° N., Hour Angle 2h 20m, or 35°.

To find tan $\beta = \cos h \cot c$

By the rule given above the latitude of Vertex will be equal to the latitude of place of observation, viz. 32°.

From the Table we have for the column headed "Latitude of Vertex 32°."

Long. from Vertex.	Latitude	Course	Distance
34°	27° 24′	72° 47′	1786′
35	27 8	72 19	1842
36	26 51	71 52	1898

The only column with which we are at present concerned is that which contains the latitude, but the extract is given in full to illustrate the completeness and comprehensiveness of a set of Tables, which, having regard to the variety of purposes to which they may be put, form the most wonderful shillingsworth of tabular matter in the world—

To find the value of β we have then to take out the latitude corresponding to "Longitude from Vertex" equal to the hour angle, viz., 35°, which is 27° 8′.

Then tan β=tan 27° 8′, which in the Table of Natural Tangents is given as .51246, the value already obtained by logarithmic work.

An additional example dealing with the celebrated star β Persei is given below.

In latitude 25° N. required the Azimuth of β Persei (*Algol*), Dec. 40° 40′ N. at an hour angle of 2h 40m. Entering Towson with Lat. Vertex equal to latitude 25°, and Diff. Long. 2h 40m (40°), we have latitude 19° 39′.

Thus

tan β = tan 19° 39′	=.357.
2 tan β	.714+
cot p_1 (49° 20′)	.859−
cot. p_2	.145−

whence $p_2 = 180° - 81° 45' = 98° 15'$

Declination $= 8° 15'$ S.

From Davis for Lat. 25° N. Dec. 8° 15′ S. at 2h 40m

Bearing N. 125° 15′ W.

And Azimuth required N. (180° −125° 15′) W. or N. 54° 45′ W.

The following Table will serve to give an idea of the extent to which the method is available under varying conditions of latitude and hour angle.

Azimuths of Circumpolar Stars.

Approximate limits of Declination for various Hour Angles at which the method is applicable.

Hr. Angle	Latitude.								
	10°	15°	20°	25°	30°	35°	40°	45°	50°
h	°	°	°	°	°	°	°	°	°
1	To 37	To 43	To 48	To 53	57 to 35	60 to 42	64 to 49	67 to 56	69 to 62
2	,, 37	,, 41	,, 47	,, 51	56 ,, 29	59 ,, 38	62 ,, 45	65 ,, 52	68 ,, 58
3	,, 34	,, 39	,, 43	,, 47	To 51	55 ,, 29	58 ,, 37	61 ,, 44	64 ,, 51
4	,, 31	,, 35	,, 39	,, 42	,, 45	To 49	To 52	55 ,, 30	58 ,, 37
5	,, 27	,, 30	,, 32	,, 34	,, 36	,, 38	,, 41	To 44	To 46

In the preceding problem the Azimuth was determined for a star of high declination, in connection with Tables based on the Time Azimuth, with Argument hour angle. It may be of interest to shew how by the same artifice of introducing the supplement of the Angle, a similar method may be employed for Tables having the altitude as Argument. The problem is rather of theoretical than practical interest, since no complete set of Altitude Tables are in common use, but it is perhaps worthy of passing notice as affording another instance of the employment of the Towson Tables in

simplifying calculations even when the right angle, to which the Tables are supposed to relate, is not directly concerned.

PROBLEM.

Having given the latitude of place, and altitude of a body outside the limit of declination tabulated, to find the declination of a body, which for the same values of latitude and altitude will have the supplement of the Azimuth of the original body.

Let c be the colatitude of place, p_1 the polar distance of the original body, z its zenith distance, and Z the Azimuth required.

Let p_2 be the polar distance of the fictitious body, which at the same altitude will have an Azimuth equal to $(180° - Z)$.

Then, from the fundamental formula for the cosine of an angle in a spherical triangle we have

$$\cos Z = \frac{\cos p_1 - \cos z \cos c}{\sin z \sin c}$$

whence $\sin z \sin c \cos Z = \cos p_1 - \cos z \cos c$ (1).

similarly $\sin z \sin c \cos (180° - Z) = \cos p_2 - \cos z \cos c$ (2).

Adding each side of equations (1) and (2)

$$\cos p_1 + \cos p_2 - 2 \cos z \cos c = 0$$

and $$\cos p_2 = 2 \cos z \cos c - \cos p_1 \quad (3).$$

From equation (3) we can determine the value of p_2, since c, z and p_1 are all known, and if p_2 happens to fall within the limits of any Tables to which we have access $(180° - Z)$, and thence Z, may be found by inspection.

Example.

In latitude 19° N. required the Azimuth of Capella (Dec. 45° 55′ N.) at an altitude of 24°.

To find the value of 2 *cos z cos c.*

Log 2	.301
L cos z (66)°	9.609
L cos c (71°)	9.513
(Sum) Log	9.423
Nat No.	.265+
L cos. p_1 (44° 5′)	.718−
(Sum)	.453−

p_2=116° 56′ and Declination is 26° 56′ S

From the Goodwin Table for Lat. 19°, Dec. 27° of *Contrary Name*, Altitude 24°, Azimuth is N. 132° 46′ W., whence the bearing of Capella is (180°−132° 46′) W., or N. 47° 14′ W.

The Solution by Towson.

In the expression

$$\cos p_2 = 2 \cos z \cos c - \cos p_1.$$

let $\cos z \cos c$ or $\cos z \sin l = \sin \beta$

then $\cos p_2 = 2 \sin \beta - \cos p_1.$

To find sin β.

Let the triangle $P\,A\,V$, as before, represent the triangle solved in Towson.

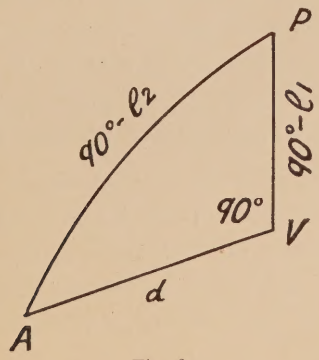

Fig. 6.

Let l_1 be the latitude of Vertex V, l_2 of the point A on the great circle, d the distance of A from the Vertex.

Then from the diagram

$$\sin l_2 = \sin l_1 \cos d.$$

Comparing this with the equation for sin β, viz.:

$$\sin \beta = \sin l \cos z$$

We see that if we take latitude of place as latitude of Vertex, and z the zenith distance in minutes as the distance d, l_2 the corresponding latitude for the point A in Towson, will be the value of the angle β.

For latitude of Vertex we have 19°, latitude of place of observation. Looking in the columns with this heading for distance from Vertex equal to the zenith distance (90°−24°) or 3960 miles, the latitude corresponding is 7° 38′, which is the value of β.

Then $\cos p_2 = 2 \sin 7° 38' - \cos 44° 5'$

Nat sin 7° 38′	.133
	2
	.266+
Nat cos 44° 5′	.718−
Nat cos p_2	.452−

$p_2 = 116° 52'$

And declination is 26° 52′ S., which is within a few minutes of the value arrived at by logarithmic calculation.

Azimuth for High Latitudes.

The problems hitherto treated have reference chiefly to Declinations beyond the ordinary limits. In that which follows the use of the Tables is extended to latitudes higher than the ordinary value of 60°, to the polar regions, for instance.

Let PP' represent the North and South Poles, S a body of Declination not greater than 23° 27′, Z_1 the zenith of the place of observation, the colatitude $P\,Z_1$ being less than 30°.

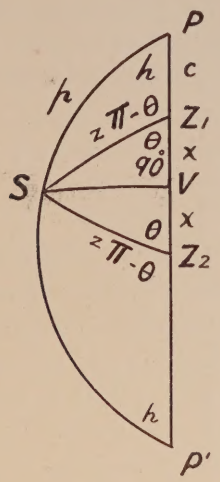

Fig. 7.

With S as centre, and $S\,Z_1$ or z, as radius, describe an arc of a small circle cutting $P\,P'$ in Z_2. From S let fall the perpendicular arc $S\,V$, dividing the triangle $S\,Z_1\,Z_2$ into two triangles equal in all respects.

It is obvious from the diagram that in the triangle $P'\,Z_2\,S$ we have the angle $P'\,Z_2\,S$ equal to the angle $P\,Z_1\,S$, the Azimuth required, but reckoned from the South Pole instead of North.

In resorting to the Azimuth Tables two cases will occur in practice, according as Z_2 is found to be in the same, or the contrary, hemisphere to Z_1. If the two points are in the same hemisphere, as soon as $P\,Z_2$ is known, we look out in the Azimuth Tables the value for latitude and declination of

Same Name, which in the diagram is represented by θ, and take the *supplement*, or $(180° - \theta)$.

If Z_2 is in the opposite hemisphere to Z_1 with the same hour angle and latitude, but with declination of *Contrary Name*, we shall find from the Tables an angle numerically equal to the required azimuth, but reckoned from the opposite Pole. In this case, therefore, we have only to change S. to N. or N. to S. in naming the Azimuth, keeping the numerical value unchanged.

Examples.

Example 1. In latitude 72° N., when the declination is 10° N., and hour angle 5h 20m, required the Azimuth.

From Table 29 Raper, or a similar Table for bodies on the Prime Vertical, we find the latitude corresponding to hour angle 5h 20m, declination 10°, to be 45.5°.

Thus $P\,V = 90° - 45.5° = 44.5°$

$P\,Z_1 = 18$

(Diff.) $x = 26.5$

$2\,x = 53°$

$P\,Z_2 = P\,Z_1 + 2\,x = 18° + 53° = 71°$, and

Latitude is 19° N.

From the Davis Tables Azimuth for Lat. 19°, Dec. 10° (of Same Name), Hour Angle 5h 20m, is N. 83° 37′ W.

Both latitudes being North we take the supplement, so that for latitude 72° N. the Azimuth is N. 96° 23′ W.

Checking our result by the Davis Tables for Latitudes 61° to 78° we have the value given as N. 96.4° W.*

Example 2. In latitude 62° N., when the declination was 12° N., and hour angle 3h 40m, required the Azimuth.

* *High Latitude Azimuths* (61° *to* 78°). By Percy L. H. Davis. J. D. Potter, 145 Minories, E.1.

Here Table 29 Raper gives for declination 12°, hour angle 3h 40m, the value of the latitude as 20° 3′.

$P\,V$	=	69.7°
$P\,Z_1$		28
(Diff.) x		41.7°
$2\,x$		83.4°
		28
$P\,Z_2$		111.4 Latitude 21.4° S.

From Davis Tables for Lats. 0° to 30°, for Lat. 21°, Dec. 12° of *Contrary Name*.

Azimuth S. 116° 16′ W.

For change of *Name* we have only to substitute N. for S. and the Azimuth required is N. 116° 16′ W.

The High Latitude Table of Davis for Lat. 62° N., Dec. 12° N., hour angle 3h 40m gives Azimuth N. 116° 4′ W.

The Ambiguous Case.

It may be of interest to point out that the artifice employed in the foregoing demonstration brings in what is known in Trigonometry as the "Ambiguous Case." In general when three of the six parts of a spherical triangle are given the triangle may be regarded as completely determined, and the remaining three may be deduced from one or other of the fundamental formulæ which connect the six parts of the triangle, divided into groups of four. If, however, the three parts given consist of two sides, and the angle opposite to the smaller of these sides, then two triangles may be found, each of which satisfies the condition of having the three parts given.

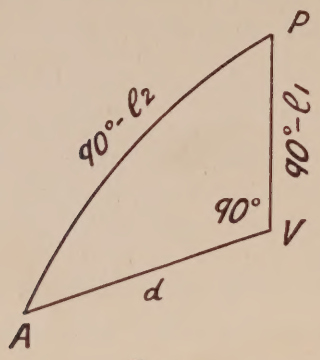

Fig. 8.

Thus in the present instance each of the triangles $P\ Z_1\ S$, $P\ Z_2\ S$ has the two parts p polar distance, z zenith distance, with the hour angle h common to both. But whereas in $P\ Z_2\ S$ the angle representing the Azimuth is $P\ Z_2\ S$, or θ, the corresponding angle in $P\ Z_1\ S$ is $(180° - \theta)$, the supplement of the former angle. It is a necessary condition for the existence of two triangles connected in this way one with the other that the side z should be *less* than the side p, so that the feasibility of the problem is limited by the condition that z should be less than p. Now since at rising or setting the value of z is 90°, while p, which is by hypothesis the polar distance of a body of *Same Name* as the latitude, is less than 90°, and as z is continually decreasing up to the meridian, while the value of p remains practically the same, there must be a particular hour angle when z the zenith distance is equal to p the polar distance, and it is only when the hour angle is less than this value that the method described becomes practicable. In the high latitudes under consideration the altitude may be expected to become equal to the

declination at an hour angle of between five and six hours. For an interval of two or three hours after this epoch of equal altitude and declination has taken place in the forenoon, or preceding it in the afternoon, the periods most suitable for observations for longitude by chronometer, the method may be employed with advantage.

CHAPTER IV.

Methods for Azimuth, other than the Azimuth Tables, in Common Use. The A.B.C. Tables—Their Origin, Rise, and Progress.

These celebrated Tables have for many years loomed so largely in the public eye that a few notes of a historical nature, extracted from a letter of the late Captain Lecky, in the *Nautical Magazine* of December, 1895, may be of some interest. According to the account there given—

"The A and B Tables first appeared in the *Nautical Magazine* for 1846, but the author's name was not given. The Tables jointly occupied less than one and a quarter pages, and were grossly incorrect.

"In the December number of the 'Nautical' for 1873, the Tables were reproduced without alteration by Captain Trivett, in connection with an article on Ex-Meridians.

"In the February number for 1874 Mr. Bolt brought out a corrected Table B—expanded to three quarters of a page—and shewed how it could be used in connection with the Equation of Equal Altitudes.

"In May of 1883, Mr. H. S. Blackburne, of the P. and O. Co., published a small pamphlet entitled 'A and B Tables for Correcting the Longitude, etc.' They occupied twelve partial pages, and were nothing more than a development of the original Tables, and Mr. Blackburne states so in his preface. The late Mr. W. H. Rosser knew that Blackburne

was working on these Tables, but this did not prevent him in July of the same year from publishing them in his *Stellar Navigation*, though in a somewhat different form.

"My own connection with the A and B Tables is related in the Preface to the 9th edition of *Wrinkles*, and again referred to on page 188 of that work; nevertheless it is as well to repeat it here. Mr. Blackburne had long urged me to insert his Tables in *Wrinkles*, and ultimately I agreed to do so on the understanding that I was at liberty to deal with them as seemed to me best. This was agreed to, and accordingly the A and B Tables were recomputed from end to end, and expanded from twelve pages to eighteen complete pages.

"These last and another are called 'Lecky's A, B, C Tables,' merely to distinguish them from their predecessors.

"Table C was first produced by Mr. A. C. Johnson, R.N., in his now famous pamphlet, which has earned for him the title of 'Cloudy Weather Johnson.' I had his kind permission to insert it in the first edition of *Wrinkles*. It was next produced by Rosser, and finally it appears in the ninth edition of my book, having increased in size from two pages to six pages. I am accountable for the enlargement."

Thus the Tables in question appear to have been connected with the names of almost all the prominent Table compilers of successive generations, except that of the original author, whose identity appears to have been shrouded in as impenetrable a cloud as that which envelops the mysterious Martelli.

The theory of the A B C method for Azimuth.

Once again we have to resort to that invaluable formula connecting four adjacent parts of the Spherical Triangle.

For the problem dealt with is the Time Azimuth, in which having given colatitude, polar distance, and hour angle, two sides and the angle included, we have to find the angle $P\,Z\,X$ adjacent to the side representing the colatitude.

With the usual notation we have once more the formula:

$$\cot p \sin c = \cot Z \sin h + \cos h \cos c$$

dividing each side by $\sin h \sin c$ we obtain

$$\cot p \operatorname{cosec} h = \cot Z \operatorname{cosec} c + \cot h \cot c$$

$$\cot Z \operatorname{cosec} c = \cot p \operatorname{cosec} h - \cot c \cot h$$

or Table C Table B Table A

$$\cot Z \sec.\ \text{lat} = \tan d \operatorname{cosec} h - \tan \text{lat.} \cot h.$$

The Tables are carried to three places of decimals, and full instructions as to their use in finding Azimuth are given in pages 423-426 of Lecky's *Wrinkles*. The number of figures employed is small, and the work involved appears delightfully simple. A good deal of trouble and thought, however, are involved in the selection of the algebraic sign appropriate to each of the three terms involved.

There seems also no particular reason for the division of the original equation by $\sin h \sin c$, unless it was done with the object of keeping the computer as much as possible in ignorance of what it was that he was about, a form of procedure which was somewhat in favour in the forties of the last century. It would seem as if the formula might well have been permitted to remain in the form in which it first occurs. Then with a small supplementary Table occupying but a few inches, a single Table might be constructed strong enough to take over the whole operation single-handed, in the following manner.

Alternative method for Azimuth.

From the fundamental formula given above we have

$$\cot Z \sin h = \cot p \sin c - \cos h \cos c.$$

Let us assume that $\cot \theta = \cos h$.

Then

$$\cot Z \sin h = \cot p \sin c - \cot \theta \sin l$$

From this formula it is evident that if we compute a Table giving the values of the expression

$$\cot a \sin b$$

and arrange the Table with one quantity, a for instance, at the top, and b at the left hand side, we shall be in a position to find the value of Z with the aid of this single Table.

But first it will be necessary to find the value of θ, the angle which has its cotangent equal to the known value of $\cos h$. To effect this without any calculation whatever we have only to turn again to the Towson Great Circle Tables, and the columns headed "Lat. Vertex 45°" will give us by inspection the values required.

For if as before l_1 is the latitude of a point at a distance of longitude from Vertex equal to L,

$$\cos L = \tan 45° \cot (90° - l_1) = \cot (90° - l_1)$$

Thus if $h = L$, $90° - l_1$ is the value of θ which has its cotangent equal to $\cos h$.

Thus we have corresponding values for h and θ:

h	θ
20°	46° 49′
25°	47 50
30	49 7

and so on—

The angle θ, it may be observed, will always follow the

sign of h, that is, if h is greater than 90°, so also will be θ, and in each case we shall deal with the supplement of the angle.

But in any case we should have the same trouble in determining the algebraic signs of the several terms by which the use of formulæ of this nature is very severely handicapped.

Working out example 1 (*Lecky's Wrinkles* p. 423) we should proceed as follows—

The formula is—

$$\cot Z \sin h = \cot p \sin c - \cot \theta \sin l$$
$$\text{where } \cot \theta = \cos h.$$

And the data are

Lat 24° S., Dec. 29° N., Hour Angle 29° 30′ (1h 58m) W.

To obtain θ we find in Towson.

Lat. of Vertex 45°	
Longitude from Vertex	Latitude
29°	41° 10′
30	40 53

Whence for 29.5° $\theta = 90° - 41°$, or 49° nearly.

The Table of products of sines and cotangents might take the form shewn below.

Sine	Cotangent	
	29°	30°
29°	.874	.840
30	.902	.866

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For cot p sin c

$p = 90° + 29° = 119°$, $c = 90° - 24° = 66°$

cot p	9.7437
sin c	9.9607
Log.	9.7044
Nat No.	.5063

For cot θ sin l

$\theta = 49°$ $l = 24°$

cot θ	9.9392
sin l	9.6093
log	9.5485
Nat No.	.3536

Thus we shall have to take the algebraic sum of the two quantities:

	.506−
	.354−
(Sum)	.860−

The values .506, .354 would in practice be obtained from the single Table, formed of the products of sines and contangents, an example of which is given above.

Entering this Table with 29.5° as hour angle in the sine column the number corresponding would be

Under 29°	*Under* 30°
.888	.853

So that the Azimuth corresponding to .860 would be S. (180° − 29° 48′) W. or S. 150° 12′ W., the supplement being taken on account of the negative sign, and the bearing required is N. 29° 48′ W.

The actual amount of work involved is much the same as in the ordinary method, and there is the same trouble with regard to algebraic signs, but it is a distinct advantage to have only one Table to handle instead of three.

The Spherical Traverse Table of Norie's Epitome.

The Spherical Traverse Table which occupies pages 522-555 of the Norie collection affords a short approximate solution of the general problem.

"Having given given two sides and the included angle in a Spherical Triangle, to find the remaining angles."

Of this type the Time Azimuth is one of the most important examples, and it is solved in the text books by the logarithmic formulæ

$$\tan\frac{A+B}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2}$$

$$\tan\frac{A-B}{2} = \frac{\sin\frac{a-b}{2}}{\sin\frac{a+b}{2}}\cot\frac{C}{2}$$

the general formula adapted to the special conditions of the "Triangle of Position."

Let us suppose that we have two triangles $A\ B\ C$, $A'\ B'\ C'$, which have sides a, b in the one respectively equal to a', b' in the other, and in $A\ B\ C$ the angle at C a right angle, while the angle $A'\ C'\ B'$ has some other given value.

Then by the formulæ quoted above, in the triangle $A'\ B'\ C'$

$$\tan\frac{A'+B'}{2} = \frac{\cos\frac{a'-b'}{2}}{\cos\frac{a'-b'}{2}}\cot\frac{C'}{2}$$

and in the triangle $A\ B\ C$

$$\tan \frac{A+B}{2} = \frac{\cos \frac{a-b}{2}}{\cos \frac{a+b}{2}} \cot 45^\circ = \frac{\cos \frac{a'-b'}{2}}{\cos \frac{a'+b'}{2}}$$

Substituting for $\dfrac{\cos \frac{a'-b'}{2}}{\cos \frac{a'+b'}{2}}$ in the first equation,

$$\tan \frac{A'+B'}{2} = \tan \frac{A+B}{2} \cot \frac{C'}{2}$$

Similarly it may be shewn that

$$\tan \frac{A'-B'}{2} = \tan \frac{A-B}{2} \cot \frac{C'}{2}$$

If, therefore, we have the values of A, B, the base angles in the right-angled triangle, we can find from the formulæ the value of $\dfrac{A'+B'}{2}$, one half the sum of the base angles for the triangle which has a given angle C' in place of the right angle. So also we can determine the value of $\dfrac{A'-B'}{2}$. Adding and subtracting we have values for A' and B' respectively.

The Method of Tabulation.

Applying the general formulæ to the special case of the Time Azimuth we shall have

Z' (Azimuth)	for	A'
X' (Angle of Position	,,	B'
p (polar distance)	,,	a' or a
c (90°—latitude)	,,	b' or b
h (Hour Angle)	,,	C'

In Part I of the Table are given the values of X and Z for each whole degree, or in some cases two degrees, of latitude

and declination from 1° to 70°, when the angle included between the two given sides is a right angle.

In using the Tables the first step is to add and subtract the two angles tabulated as X, Z, naming the quantities resulting in Sum (A) and Diff. (A) respectively.

It remains to convert $\frac{Z+X}{2}$, $\frac{Z-X}{2}$ into $\frac{Z'+X'}{2}$, $\frac{Z'-X'}{2}$.

This conversion is effected by Part II of the Table, which is styled the "Conversion Table," the nature of which will be best described by means of a practical example.

In latitude 64° N., when declination is 30° N., at an hour angle 4h 24m, we wish to find the azimuth.

Entering Part I with latitude 64° N., declination 30° N., we have the values for the base angles of the right-angled triangle $Z=75°\ 48'$, $X=29°\ 23'$.

Adding these angles we obtain the approximate value of 105°, which as already mentioned is styled Sum (A). The object of the Conversion Table is to convert by a process of inspection Sum (A) into $\frac{Z'+X'}{2}$ which is called Sum (B).

To obtain by calculation, $\tan \frac{Z'+X'}{2}$ *i.e.*, tan (Sum B) from the known values of $\tan \frac{Z+X}{2}$, we employ the general formula given above, as adapted to the Triangle of Position, viz.

$$\tan \frac{Z'+X'}{2} = \tan \frac{Z+X}{2} \cot \frac{h}{2}$$

which may be written

tan (Sum B) = tan (one half Sum A) cot (one half hour angle)

The work by logarithms for the approximate values of the base angles 76° and 29° will then be as follows:

Z	76°	Log tan one half (Sum A) 52° 30′	.115
X	29	Log cot one half hour angle (33°)	.187
(Sum A)	105°	Log tan $\frac{Z'+X'}{2}$	.302

$$\frac{Z'+X'}{2} \text{ or (Sum } B)=63° 29'$$

The Conversion Table spares us all this logarithmic work, so that when, by merely adding the base angles, we have obtained Sum (A), we have only to turn to the Table for its conversion to Sum (B) by inspection.

Thus in the present instance we find the entries.

SUM (B) OR DIFF. (B).

Sum (A) or Diff. (A)	Included Angle
	4h 24m
104°	63.1°
106	63.9°

the necessary calculations having been carried out in advance, and embodied in the "Conversion Table."

For 105° we take the value midway between these for 104° and 106°, or 63.5°, agreeing with the value found above by calculation of 63° 29′.

Similarly for Diff. (B) subtracting the values given above of Z 76°, X 29° we have for Diff. (A) the value 47°. For Diff. (B) we obtain from the Table the value 33.8°

SUM (B) OR DIFF. (B).

Sum (A) or Diff. (A)	Included Angle
	4h 24m
47°	33.8°

We are now in a position to conclude the work as follows:

Z	76°		
X	29		
Sum (A)	105°	Sum (B)	63.5°
Diff. (A)	47	Diff. (B)	33.8
		Z'	97.3
		X'	29.7

The greater value being taken for Z' because Z is greater than X in the Table.

Latitude being North the Azimuth is reckoned from the North Pole, so that the bearing required is N. 97.3° W. or S. 82.7° W.

Two important advantages are claimed for this method, the first, that algebraic signs are entirely dispensed with, the matter being simply an affair of addition and subtraction throughout. Neither have we to discriminate in the end as to the quadrant in which the resulting angle is situated, since, should it lie in the second quadrant, the fact is indicated by a value greater than 90° in the result. Again, by a single process, both of the base angles are determined at once, so that the Tables can be used with great facility for finding by a single operation the first and last courses on a Great Circle, a great deal of unnecessary labour being saved thereby.

*The Equatorial Azimuth Table.**

This Table, a complete Azimuth Table for latitude 0°, offers a simple and universal method of finding the Azimuth, either by Time or Altitude Azimuth, applicable in all

* *An Equatorial Azimuth Table.* By H. B. Goodwin. J. D. Potter, 145 Minories, E.1. Price 10s.

latitudes, to bodies of any declination North or South. The fundamental notion upon which the method is based is simply this, that for any heavenly body observed in latitude $l°$ a second body may be found, which, if observed at a place on the Equator, would have the same hour angle and Azimuth as that actually observed in latitude $l°$. This is effected by means of the same "Conversion Table" that is employed in connection with the Spherical Traverse Table of the Norie Epitome, and the actual procedure is as follows. Having given p_1 and z_1 the polar distance and zenith distance of the body observed, by means of the Conversion Table, these are converted respectively into p_2 and z_2 the polar distance and zenith distance of the imaginary body which observed at the Equator would have the same hour angle and azimuth. It then only remains to look out in the Azimuth Table the bearing corresponding to the polar distance so obtained.

The full procedure will be as follows:

Write down the values of p_1 and z_1, the polar distance and zenith distance of the body, and take their Sum and Difference, marked Sum (A) and Diff. (A) as before.

Convert Sum (A) and Diff. (A) into Sum (B) and Diff. (B) as usual, the Argument of the Conversion Table in this case being latitude in place of hour angle.

Adding and subtracting the values obtained we have p_2 and z_2, the values appropriate to the Equator, and the value of the Azimuth is looked out in the Equatorial Table accordingly.

As an illustration we may take the example given above for latitude 64° N., the full details being as shewn below.

In latitude 64° N. the altitude of a star of declination 30° N. was 37° 8′ at 4h 24m p.m. Required the Azimuth.

$p_1 = 60.0°$

$z_1 = 52.9$

Sum (A)	112.9	Sum B)	81.3°
Diff. (A)	7.1	Diff. (B)	15.0

p_2 96.3

z_2 66.3

As we are working from the Pole of same name as latitude, viz., North, the value of 96.3° for polar distance indicates declination 6.3° S. In the Equatorial Azimuth Table for the values of hour angle and declination we find the entries which follow:

Zenith Distance and Azimuth for Latitude 0°.

Declination	Hour Angle 4h 24m	
	Z D	Azimuth
6°	66.1°	83° 26′
7	66.2	82 21

The Azimuth required therefore for declination 6.3° is approximately S. 83° W., as found in the first instance by the Spherical Traverse Table, the value of 83° being reckoned from the South Pole because at the Equator the Azimuth follows the Name of the declination. More complete particulars as to the theory upon which this method is based are given in an Appendix to the Tables.

CHAPTER V.

Application of the Azimuth Tables to the solution of various Problems on finding Great Circle Courses by the Azimuth Tables.

It is somewhat remarkable, having regard to the fact that the Azimuth Tables, within certain limits as to sides and angles, present a general solution of a particular case of the solution of the spherical triangle, that more than thirty years from the first Burdwood publication were suffered to elapse before it was pointed out that the Tables might be profitably employed for purposes quite distinct from those for which they were in the first instance designed, more particularly with regard to the calculation of Great Circle Courses. The use of the Tables for this purpose was proposed by the present writer in a pamphlet entitled "The Ex-Meridian treated as a Problem in Dynamics" published by Messrs. George Philip & Son in year 1894. So obvious a use of the Tables had very probably occurred to others earlier, but, so far as the writer is aware, no reference to such processes had previously appeared in any publication.

The pamphlet being now out of print it may be as well to reproduce here the suggestion in the form in which it was first presented.

"Other applications might easily be found, but perhaps one of the most striking lies in their adaptation to the problem of finding the successive courses to be followed

in order to keep a ship upon a given great circle, passing, suppose, through two ports A and B, when the port for which the ship is bound happens to be within the tropics.

Here the two sides are represented by the arcs $P\,A$, $P\,B$, joining the two places with the North or South Pole, while the included angle is their difference of longitude. Let the course required be the initial course from A to B.

In using the Tables the latitude of A will be looked out as latitude, the latitude of B will be the Declination, the difference of longitude will be the hour angle.

Let us suppose, for instance, that it is required to find the first Course from Cape Horn (Lat. 55° 59′ S., Long. 67° 14′ W.) to St. Helena (Lat. 15° 55′ S., Long. 5° 54′ W.).

With latitude 56°, declination 16° (Same Name) and hour angle 4h 5m, we have azimuth 105° 12′. Whence the course required is N. 74° 48′ E. The question is given as an example in Raper, and the calculated result is N. 75° 2′ E. As has been said, the rule holds good so long as the port for which the ship is bound is situated within the tropics.

In other cases it falls outside the limits of the Tables, but the difficulty which consequently arises is easily overcome.

Since every circle cuts the Equator in two points, and therefore passes through the tropics, we have only to determine by calculation the position of some point on the circle, between Lat. 23° N. and Lat. 23° S., not less distant than 1800 miles from the point of departure, and the successive courses to this point, and therefore to B, for given differences of longitude, may be taken one by one from the Tables. A point on the Equator is preferable, since its determination is the most simple, and therefore affords least trouble.

The limitation as to a distance not under 1800 miles, it may be observed, was rendered necessary by the fact that the original Burdwood as first issued had a limit of 60° of altitude, so that when the zenith distance fell below 30°, or 1800′, no values of the Azimuth were recorded. Now that, as in *Brown's Completed Burdwood*, values are continued right up to the meridian, this limitation has been removed.

The course might have been found with almost equal simplicity by means of the Norie Spherical Traverse Table as follows:

From Table I for Lat. 56°, Dec. 16° (Same Name)

Z	80°	53′			
X	35	3			
Sum (A)	115	56	Sum (B)	69.7°	
Diff. (A)	45	50	Diff. (B)	35.6	
			Azimuth	105.3	

S 105° 18′ E. or N. 74° 42′ E.

Working as we do with the Spherical Traverse Table to the nearest decimal of a degree, the process gives a result even closer to the truth than the Azimuth Tables, and for latitudes up to 70° the process is perfectly general.

The Azimuth Tables and the Ex-Meridian.

The notion of employing the Azimuth Tables in connection with the reduction of an altitude to the meridian was also suggested for the first time in the pamphlet of 1894, an extract from which is given above. The principle of the method there proposed may be gathered from the quotation which follows: "The Ex-Meridian is then, for the most part, an observation limited to a range of one or two points from the meridian, and it is worked either by means of an

approximate formula, based upon the assumption that the bearing is small, or more commonly perhaps in these days, by means of Tables giving by inspection the results of this formula."

The most complete set of such Tables is that compiled by Messrs. Brent, Walter, and Williams, published in 1886, which gives values of the reduction up to 70° of declination, so that the Tables are available not only for Sun, Moon, and Planets, but for Stars also.

The Tables are constructed upon the approximate formula given in Godfray's Astronomy, as follows:

$$\text{Reduction} = \frac{\cos \text{lat.} \cos \text{dec.} \sin^2 15'}{2 \sin \text{mer zen dist.} \sin 1''} h^2$$

where h is the hour angle in minutes of the body observed.

Thus the expression given above is the motion in altitude of the body during h minutes, and may be regarded as an arc described in a time h.

But if a body were describing a circle with a uniform acceleration of f'' per minute, the arc described in h minutes from rest would be given by the expression

$$\tfrac{1}{2} f'' h^2$$

Comparing these two expressions, we see that since the latitude, declination, and zenith distance are practically constant quantities, the body may be regarded as moving in altitude with a constant acceleration of f'' per minute, where

$$f = \frac{\cos \text{lat.} \cos \text{dec.} \sin^2 15'}{\sin \text{mer zen dist.} \sin 1''}$$

Now when a body is upon the meridian, its motion is at right angles to the meridian, and its velocity in altitude is consequently zero. And when it attains an hour angle of h

minutes, its rate of change of altitude $d\,z$ is given by the well known formula

$$d\,z = \sin A\,z \cos \text{lat}\; d\,h$$

where $d\,h$ is the unit of time.

So that the mean velocity during the time h, which is one half the sum of the velocities at the beginning and end, is

$$\tfrac{1}{2}\,(0 + d\,z)$$

And by the principle alluded to above that

$$\text{Space} = \text{mean velocity} \times \text{time}$$

we have Reduction to Meridian $= \tfrac{1}{2}\,d\,z\,h$

or, stated in words,

Reduction = one half rate of change of altitude at hour angle h multiplied by the number of minutes of time from the Meridian.

Thus the formula obtained would be represented by

$$d\,z = \sin Az.\ \cos l\; d\,h$$

where $d\,h$ is half a minute of time. So that $d\,z$ = change of alt in half minute $= \sin Az.\ \cos.\ l\ (30\text{s} \times 15)$

$$= \sin Az.\ \cos l\ 450''.$$

The following practical example from Riddle was employed to illustrate the working of the method.

Date July 12th. Approximate latitude 12° S., time 0h 8m 32s, declination 22° 6′ N., corrected altitude 55° 54′.

From the Davis Tables for latitudes 0° to 30° Azimuth is about N. 3° 30′ W.

From the Traverse Tables, with 450 for Distance and 3° 30′ as Course we have Departure = 28 nearly.

$$\text{Thus } d\,z = 28 \cos l.$$

Entering the Traverse Table again, with 28 as Distance and 12°, the latitude, as Course, Diff. Lat. is 27.4.

Then

$$\text{Reduction} = 27.4'' \times 8.5 = 3'\ 53''.$$

In the year 1894, the date of the publication of the pamphlet, the practical utility of the method of employing the Azimuth was materially limited by the circumstance previously mentioned that in none of the Azimuth Tables were values given when the altitude exceeded 60°. It seemed therefore at the time hardly worth while to supply a special Table giving values for the product

Sin *Az* cos *lat*

and the method of employing a double entry of the Traverse Table as in the example worked above was suggested. Some ten years after, however, a Table of Supplementary Azimuths* computed by Mr. Percy Davis was published, in which approximate values are given for the missing Azimuths right up to the Meridian. The publication of this work rendered the use of the Azimuth in connection with the Ex-Meridian problem much more general, and in the year 1903 a second little work entitled "A New Table for Solving the Ex-Meridian Problem on Kinematic Principles" was brought out, giving in tabular form the "Rate of Change of Altitude in Minutes, per minute of Time;" in other words, supplying the product of

Sin *Az* cos *lat*

The form of this Table is shewn by the extract which follows—

Azimuth	Latitude			
	39°	39° 30′	40°	40° 50′
3° 0′	.610	.606	.601	.597
30	.712	.707	.702	.696
40	.813	.807	.802	.796
30	.915	.908	.902	.895

* *Davis's Ex-Meridian Tables with Supplementary Azimuths.* J. D. Potter, 145 Minories.

While the method of use is illustrated by the practical example given below.

Latitude by account 40° S., true altitude 59° 48′, hour angle 14m 40s., declination 9° 54′ S. Required the Reduction.

From Burdwood, for lat. 40°, declination 10° (Same Name), one half of hour angle 7m 20s, we obtain Azimuth 3° 30′.

From Table, for latitude 40°, Azimuth 3° 30′ we have rate .702.

$$\text{Reduction} = .702 \times 14.7 = 10.3'.$$

This second pamphlet, with the Table giving "rates of change of altitude," as already mentioned, was published in 1903. By one of those coincidences of which the history of Navigation offers many instances, Captain Armistead Rust, of the United States Navy, some five years later independently hit upon the same idea of utilizing the Azimuth for the solution of the Ex-Meridian problem and in the preface to his "Ex-Meridian" was good enough to include the little brochure containing the Table in his list of works* consulted. "This last book," wrote Captain Rust, "was received as these Tables were going to press, and I find that Table V is practically the same as the Table given by Mr. Goodwin.

. . . . . .

Mr. Goodwin's pamphlet will be found a most useful addition to the library of every Navigator."

The pamphlet unfortunately can no longer be obtained, but the Tables have found a new home in that small encyclopaedia of nautical information *Brown's Nautical Almanac.*

* *Ex-Meridian, Altitude, Azimuth and Star-Finding Tables.* By Lieut.-Commander Armistead Rust, U.S.N. New York: John Wiley & Sons. London: Chapman & Hall, Ltd.

The Mean Rate of Change applied to find Time of Maximum Altitude.

The principle of the mean rate of change of altitude may be applied with considerable advantage to an extension of the Ex-Meridian problem which the developments of recent years have brought prominently into public notice.

This extension has its origin in the largely increased, and ever increasing speed of the ocean steamships, and may be enunciated briefly as follows:—

Let us suppose that a ship is proceeding southward at a high rate of speed. An observer, taking the meridian altitude of a heavenly body, will find that the phenomenon known as the "dipping" of the body, which generally occurs when the body is upon the meridian, will take place some minutes earlier or later than the time computed for the meridian passage.

Let us suppose that the body lies to the south of the observer, and that this is also the direction towards which he is proceeding.

Then, as the observer moves southward, his horizon is shifted at a rate equal to his speed, and is, so to speak, withdrawn from the body, which continues therefore to increase its altitude.

The observation taken is therefore off, instead of on, the meridian; and it is a matter of some importance to know the time when the maximum altitude occurs, and so determine the correction to be applied to the altitude in order to reduce it to the meridian.

The same effect is, of course, produced by any change in the declination of the body, and the rate of the ship's southing (which we shall indicate by K) may be considered as the rate

compounded of the change in the ship's latitude in miles per hour, and of the motion of the body in minutes of declination per hour, or, which is the same thing, in seconds per minute.

Except in the case of the Moon, however, and perhaps of the Sun when near an Equinox, the change of declination is hardly worth taking into account.

Reverting to the typical case first selected—that, namely, of the observer approaching the body—we see that the body will continue to rise so long as the southing of the ship is greater than the rate of change in altitude.

When the velocity of the body's motion in altitude, expressed in seconds per minute, is equal to the velocity of the ship in miles per hour, we obtain the time of the maximum altitude.

Let K then represent the speed of the ship in knots. Therefore K' per hour, or K'' per minute, is the motion of the observer's horizon.

Hence K'' per minute is the rate of the body's motion in altitude at the instant of the maximum altitude.

We have then to find the time when the change of altitude per minute is K''.

For this we have the formula—

$$dz \text{ or } K''=f''\ t.$$

That is, velocity=acceleration multiplied by time. But, as has been shewn, when a body is near the meridian

$$f = \frac{\text{Cos lat cos dec sin}^2\ 15'.}{\text{Sin mer zen dist sin } 1''}$$

In the Norie Tables * (pp. 161-168) one half of this quantity is tabulated under the head of C.

Therefore $f''=2\ C$.

* *Nautical Tables.* By J. W. Norie, London: Imray, Laurie, Norie, & Wilson, 156 Minories, E.1.

And our formula becomes

$$K'' = 2\,C''\,t.$$

$$\text{Whence } t = \frac{K}{2\,C}.$$

Having now the time, we may if we please reduce the body to the meridian by the ordinary process.

Or we may proceed thus:—Since

$$\text{Reduction} = \text{Mean Velocity} \times \text{Time}.$$

$$R = \tfrac{1}{2}\,K\,t.$$

Substituting for t in terms of K, C,

$$R = \frac{K^2}{4\,C}$$

From the formula

$$t = \frac{K}{2\,C}, \; R = \frac{K^2}{4\,C}$$

it appears that both the time and reduction for a given value of K vary inversely as C, and this suggests an exceedingly simple form of Table which, used in conjunction with the Brent Table III., will suffice for all cases.

A few terms of such a tabular arrangement are shown below, a more complete Table, for values of K from 8 to 35, being given later. The Table is constructed on the assumption that C is unity, and shows the Time, and also the Reduction, corresponding to each integral value of K for this value of C.

Value of K.	C=1.	
	Time	Reduction
	Min.	
15	7.5	56″.25
16	8	64″
17	8.5	72″.25
18	9	81″
19	9.5	90″.25
20	10	100″

To find the Time and Reduction for other values of C, we have simply to divide the quantities furnished in the Tables by C.

To determine the sign of the time of maximum altitude, that is, whether the maximum takes place before or after the time of transit, a little consideration will furnish rules for our guidance.

Let us suppose that the ship is approaching the body. Then the effect of ship's motion is to increase the altitude. Therefore when the body has reached the meridian, and the motion in altitude should be zero it is apparently rising.

That is, the maximum altitude does not occur till after transit.

Similarly, when the ship is receding from the body, the latter, when on the meridian, is apparently falling, that is, the maximum occurred before transit.

When in addition to the way of the ship, the proper motion of the body in declination is worth taking into account, as in the case of the Moon, or of the Sun near a solstice, then K is the velocity compounded of the two.

First let us suppose they are moving in the same direction, as, for instance both in the direction of the bearing of the body near the meridian to the south. Then the motion of the ship southward tends to increase the body's altitude, while the change of declination southward diminishes. To find K, therefore we have the rule, for motion in the same direction *Subtract*. If in opposite directions *Add*.

A complete Table for K up to the value of 35 is appended.

Value of K.	C=1.	
	Time	Reduction
	Min.	
8	4	16″
9	4.5	20″.25
10	5	25″
11	5.5	30″.25
12	6	36″
13	6.5	42″.5
14	7	49″
15	7.5	56″.25
16	8	64″
17	8.5	72″.25
18	9	81″
19	9.5	90″.25
20	10	100″
21	10.5	110″.25
22	11	121″
23	11.5	132″.25
24	12	144″
25	12.5	156″.25
26	13	169″
27	13.5	182″.25
28	14	196″
29	14.5	210″.25
30	15	225″
31	15.5	240″.25
32	16	256″
33	16.5	272″.25
34	17	289″
35	17.5	306″.25

Practical Rules for Use of Table.

(1) By means of the Traverse Table, find hourly change of latitude due to speed of ship. In the case of all bodies except the Moon this quantity may be regarded as K.

(2) For the Moon take from the *Nautical Almanac* in seconds one tenth the change of declination for ten minutes, and apply the quantity so obtained to the change of latitude, subtracting when motion of ship and change of declination have Same Name, adding when Contrary Names. This sum or difference represents K.

(3) From the Norie Ex-Meridian Table take out C for the given latitude and declination.

(4) To find the Time and Reduction enter the special Table with argument K, and divide the quantities taken out by the value of C.

(5) For the Maximum altitude, if ship is approaching body, the maximum occurs later than time of transit, if receding from the body, earlier.

Practical Example of the Use of Table.

For an illustration of the use of the Table we may take an example from the latest *Admiralty Manual of Navigation*, Vol. II., page 117, which runs as follows:—

Find the interval of Time elapsing between the moment of meridian passage, and of maximum altitude for a ship in Lat. 30° N. steaming 180° at 20 knots. The Declination is 10° N.

The work in the Manual is given as follows:

$$\text{Sin } h = \frac{\sin Z\,X \sin A}{\sin P\,X} = \frac{\sin Z\,X}{45 \cos L \cos D}$$

h is small, and we can consider $Z\,X$ to be the meridian zenith distance $(L-D)$.

$$\text{Hence } \sin h = \frac{\sin 20^\circ \sec 30^\circ \sec 10^\circ}{45}$$

L sin 20°	9.5340
Log sec 30°	.0625
,, sec 10°	.0066
	9.6031
Log 45	1.6532
L sin h	7.9499

h=2m. 2s.

Worked by means of our Table we shall have for C from the Norie Tables, p. 161, for Lat. 30°, Dec. 10° (Same Name).

$$C = 4.9''$$

From Table for $K=20$ we have 10m.

$$\text{and hour angle} = \frac{10\text{m.}}{4.9} = 2.04\text{m.} = 2\text{m } 2\text{s.}$$

Did we wish to find the Reduction to meridian we should obtain from the Table for $K=20$ the value $100''$.

$$\text{The } R = \frac{100''}{4.9} = 20'' \text{ nearly.}$$

The Azimuth Tables and the Equation of Equal Altitudes.

Now that the old fashioned Double Altitude, the Longitude by Lunar Distance, and other problems dear to the hearts of the old-time navigators have been swept ruthlessly away, it would not be very surprising, having in view the resources of wireless telegraphy, and the increased numbers of Time Signal stations, if the processes for error of chronometer by shore observations were shortly to be added to the list of the proscribed. From the fact, however, that in Vol. II. of the latest official Manual no less than ten pages are devoted to a discussion of the Equal Altitude problem alone, it would seem that the time is not yet.

Of the two methods in common use, viz., by "Absolute Altitudes," that is, by observations on one side of meridian only, or by "Equal Altitudes" observed A.M. and P.M., the latter is for accuracy undoubtedly to be preferred. If for no other reason because, by the double observation, the "effects of personal error" and of imperfections in the instrument are greatly reduced. Unfortunately in the case of the Sun, which is in general the most convenient heavenly body for the purpose, the problem is complicated by a troublesome

correction, small in amount, but nevertheless indispensable to the necessary accuracy, which is called the "Equation of Equal Altitudes."

To obtain this "Equation" all the proofs commonly given in the text books observe much the same procedure, that is, by a simple geometrical construction they obtain the following expression

$$e=\frac{dp}{15}\cot P\,X\,Z\sec D.$$

where e is the required "Equation," dp the change of polar distance for one half the interval, $P\,X\,Z$ the "Angle of Position," and D the value of the declination.

Actually no fresh construction is required at all. For in nearly all the text books we have a proof of the formula

$$dh=\cot P\,Z\,X\sec L\,dl$$

where dh is the error in hour angle consequent on an error dl in the latitude L, when polar distance and zenith distance are supposed constant.

It follows by symmetry that when colatitude and zenith distance are regarded as constant

$$dh=\cot P\,X\,Z\sec D\,dp.$$

It is at this stage that the mathematician enters the field, and, after piloting the student through a little maze of transformations, leaves him with a complicated expression involving two terms A and B, each of which requires very careful attention in the matter of algebraical sign, with consequent risk of error.

In the explanations which follow an effort will be made to shew that the necessary "Equation" may be obtained with less trouble if the simple expression for the "Equation" given above is left unaltered in the form in which it first

occurs, more particularly when the aid of the Azimuth Tables is brought into requisition. For the value of e being never more than a few seconds of time, an approximate value for the angle $P\,X\,Z$ is all that is required, and this under existing conditions can always be obtained from one or other of the Azimuth Tables.

The process necessary may be reduced to the form of a practical rule as follows.

From the three sides $P\,X$, $P\,Z$, $X\,Z$ of the triangle of position calculate the angle $P\,X\,Z$.

Or alternatively, from $P\,X$, $P\,Z$, and one half the interval in hour angle between observations obtain an approximate value of $P\,X\,Z$ from the Tables.

Write down L sec D, D being the declination, L cot $P\,X\,Z$, and the logarithm of m (change of declination for one half the interval).

The sum of these logarithms, less 20, gives the value of the "Equation" in seconds of time.

With respect to the algebraic sign.

For polar distance, measured from the elevated Pole, increasing, *add* the equation, for polar distance decreasing *subtract*. In the somewhat unusual case, only possible when declination is of Same Name, and greater than latitude of place, the angle $P\,X\,Z$ may be greater than 90°. In that case the cotangent is negative, and the rule for signs is therefore reversed.

Applying this process to the example worked on pages 210-213, Vol. II. of the 1922 Manual, the salient features of the observations will be as follows:

At Zanzibar, 28th April, 1921.—Lat. 6° 10′ S., Long. 39° 11′ E. Half interval 3h 28m (nearly) zenith distance

55° 30′. Declination 14° 3′ 50″ N., increasing 47.4″ per hour. Required the "Equation."

PX	104°	4	L cosec	.01322
ZX	55	30	L cosec	.08401
	48	34		
PZ	83	50		
	35	$16\frac{1}{2}$	L Hav.	4.48133
	132	$24\frac{1}{2}$	L Hav.	4.96140
				9.53996
			PXZ	72° 9.

To find dp change of polar distance.

One half interval=3h 28m or 3.47h nearly, and $dp=\frac{3.47\times 47.4''}{15}$=11s nearly.

L sec D	(14° 4′)	10.0132
L cot PXZ	(72° 9′)	9.5079
Log. 11		1.0414
		20.5625

Equation+3.65s.

The sign being + because in April the polar distance, as measured from the South Pole, is increasing.

A large proportion of the above work, it will be noticed, consists in the logarithmic calculation of the angle PXZ. With the resources at our command to-day the value of PXZ can in general be found with sufficient exactness from the Azimuth Tables.

Here we have given for Declination 14° N., for latitude 6° S., in each case approximately.

The latitude being not greater than 23°, the ordinary

Davis Azimuth Table, declinations for 0° to 23°, will suffice to find the angle $P\,X\,Z$, according to the following rule.

Latitude and Declination of *Contrary* Name.

Enter Table with Declination as Latitude, Latitude as Declination, of *Contrary* Name, and half Interval as Hour Angle.

The angle $P\,X\,Z$ is 180°—Azimuth from Table.

Here Latitude is 14°, Declination 6° (of Contrary Name), one half interval 3h 28m.

Azimuth from Table is 107° 40′, and $P\,X\,Z$ is 180°—107° 40′, or 72° 20′, as compared with 72° 9′ obtained by logarithmic computation.

This method of dealing with the "Equation" directly in a single term by means of the angle $P\,X\,Z$ was first proposed in the pages of the *Nautical Magazine* for the month of November, 1889, and the process of obtaining $P\,X\,Z$ by means of the Azimuth Tables, for latitudes within the tropics, was pointed out at the time. Since that date the increased range of the Azimuth Tables has rendered the method perfectly general up to quite high latitudes.

The following example from Raper, art. 806, p. 291, may serve as an illustration of this.

The details of the observations are:

July 24th 1878.—Latitude 55° 1′ N., Longitude 0° 24′ W., Declination 19° 53′ (decreasing 31.6″ per hour). Approximate half interval between observations 5h 1m, zenith distance 65°.

The ordinary Davis Tables to 23° of declination will no longer answer our purpose for finding $P\,X\,Z$. We must have recourse to another volume of Tables, due to the same indefatigable computer who was joint author of the

original tables, which deals with latitudes 0° to 30° N. and S., declinations 23° to 64°.*

Entering these Tables with latitude 20°, declination 55° (Same Name) we have for the Azimuth the value 37° 36′.

For change of declination

$$\frac{1}{15}5 \times 31.6'' \text{ gives } 10.53''$$

Then

$L \sec D$	(19° 53′)	10.02669
$L \cot P\,X\,Z$	(37° 36′)	10.11345
Log 10.53″		1.02243
	(Sum)	21.16257
	Equation	+14.5s.

the sign + being taken because in July the polar distance measured from the North Pole is increasing.

The Azimuth Tables and the Gnomonic Charts.

In the year 1914 an important new departure was taken by the Hydrographic Department of the Admiralty in the issue of an admirable series of charts constructed upon the Central or Gnomonic projection, which have this remarkable feature that every Great Circle is represented by a straight line. The use of these charts in laying off points on a Great Circle track is sufficiently obvious, since, if a line connecting two points given in latitude and longitude be drawn, the longitude of a point where the track cuts any given parallel may be read off at sight, or alternately the latitudes of points where the line intersects given meridians may be determined with equal ease. But in addition to this primary use of

* *Sun's True Bearing or Azimuth Tables.* Between the Parallels of Latitude 30° N. and 30° S., Declinations 23° to 64°. By Percy L. H. Davis. J. D. Potter, 145 Minories, E.1.

these charts, they play also a very useful part in enabling us to readily find Azimuth for bodies of declinations 25°, 30°, etc., outside the Declination limit for Tables in common use.

Having given the ordinary data of the Time Azimuth, viz., latitude of place, declination of body, and hour angle, the problem may be solved as follows:

Upon the central meridian on which the chart is developed mark the point corresponding to latitude of place.

Take the difference between the longitude of this meridian and the hour angle of body, expressed in arc. Apply this difference to the longitude of the central meridian, and mark the resulting magnitude upon the parallel corresponding to the declination of body, thus obtaining a second point on the chart.

The line joining these two points gives us a line upon which all bodies have the same Azimuth.

To find the value of this Azimuth we have only to note the longitude of the point in which this line, produced if necessary, cuts a parallel of declination 23° or less. By comparison of this longitude with that of the central meridian we obtain the hour angle for which a body of declination equal to the latitude of the parallel employed will possess an Azimuth equal in value to that sought. This value, since the declination is not greater than 23°, can then be obtained from the ordinary Burdwood or Davis Tables.

For practical examples of this use of the Gnomonic Tables the reader is referred to Appendix (B) to the second edition (1922) of *Brown's Completed Burdwood Azimuth Tables.*

CHAPTER VI.

NOTES ON CERTAIN LEADING FROBLEMS OF NAUTCIAL ASTROMONY. SOME "ALL HAVERSINE" FORMULÆ FOR THE SOLUTION OF TRIANGLES. PROOFS OF "SECOND DIFFERENCE" FORMULÆ FOR THE EX-MERIDIAN PROBLEM FOR BALL'S ALTITUDE AND HOUR ANGLE TABLES, FOR REDUCTION TO PRIME VERTICAL, ETC. THE COVTINHO SHORT METHOD FOR ZENITH DISTANCE. THE HORARY TABLES OF BLACKBURNE AND DAVIS, AND OF THE U.S. HYDROGRAPHIC DEPARTMENT. FORMS FOR "SECOND DIFFERENCE" TABLES EXTENDING THE RANGE OF THESE TABLES.

A somewhat important new notion is presented by the Admiralty Manual of 1922, in the introduction of a new method of calculating both hour angle and Azimuth by means of the same formula which has for many years been employed in the problem of finding zenith distance for a given hour angle, viz.:

$$\text{hav } z = \text{hav } (p-c) + \sin p \sin c \text{ hav } h.$$

which, for the purpose of finding hour angle, becomes

$$\text{hav } h = \left\{\text{hav } z - \text{hav } (p-c)\right\} \text{ cosec } p \text{ cosec } c.$$

where h represents hour angle, z zenith distance, p polar distance of body, and c colatitude of place.

From this development it follows that three important problems, viz., the calculation of latitude by Ex-meridian, of hour angle from zenith distance, and of zenith distance

from hour angle, can all be dealt with by the use of one and the same formula. With the further advantage, that the Table of halved Log haversines, only needed in the solution of a triangle in which from three sides given it was required to find an angle becomes no longer necessary, while the actual amount of labour in computation is no greater in one case than the other. The method of the Manual follows closely the lines of a process suggested in the *Nautical Magazine* of August, 1899, simplified to some extent by the Table of Combined Natural and Logarithmic Haversines, which in these days forms a leading feature of the Inman, Norie, and Bowditch Tables. A similarly arranged Table of Versines, compiled by the Rev. W. Hall, who first suggested this form of Table in the pages of the *Nautical Magazine*, forms an Appendix to the later editions of *Raper's Practice of Navigation*.

In the year 1910 it occurred to the present writer that the problems dealt with might be further simplified by throwing the fundamental formula into a shape requiring but one function of the angle, viz., the haversine, and expressions for this purpose were given in papers contributed to the *Nautical Magazine* for the months of July and August in that year, the general formula being

$$\text{hav } z = \text{hav } (p-c) + \left\{\text{hav } (p+c) - \text{hav } (p-c)\right\} \text{ hav } h.$$

The formula also formed the subject of a contribution to the Proceedings of the United States Naval Institute (Vol. 36, No. 3, whole number 135). Four years later a Table of combined Natural and Logarithmic haversines was included in the official *American Practical Navigator*, and the method for zenith distance by haversines only, as given above, with a form for facilitating calculation, was included in that work.

Again a few years later, in the U.S. Naval Institute Proceedings for November, 1917 (Vol. 43, No. 11) attention was once more directed to the subject in a paper by Mr. G. W. Littlehales, a member of the U.S. Hydrographic Department, well known for his originality in devising new processes in Navigation and Nautical Astronomy.

In the course of the paper Mr. Littlehales remarked: "The feasibility thus disclosed of framing a Nautical Astronomy in which all requirements will be sub-served by a single trigonometrical Table, like the Table of Haversines, No. 45 in the *American Practical Navigator*, invested the subject with interest, because this formula, if successfully represented in graphical form, might provide the aerial navigator with the equivalent of a volume of Nautical Astronomy, in a form simple enough to fulfil the instant needs of flight."

Mr. Littlehales noticed that the general formula represented the equation to a straight line, whence he deduced this remarkable fact, that, by means of a chart so constructed that its sides were graduated to a scale of Natural haversines from 0° to 180°, we could obtain a graph in the form of a straight line, which for given values of p and c would supply the value of h for a given z, or conversely of z for a given h, with the aid only of a ruler and a pair of compasses. *

By means of this chart any spherical triangle of which the sides are given can be solved approximately for the angles, or again if two sides and the angle included are given the third side can be found.

The chart itself is simply a square diagram, and consists of

By permission of the U.S. Hydrographic Department a chart of this kind has been published by Mr. J. D. Potter, 145 Minories, E.1. Price two shillings and sixpence.

a sort of network formed by straight lines mutually at right angles, which represent a series of Natural Haversines from 0° to 180°, or from 0h to 12h.

Two problems in Nautical Astronomy which can be solved graphically by means of the diagram are the determination of hour angle and azimuth, the three sides p, c, z, being given in each case.

The Process for Hour Angle.

Mark off on the left hand upright of the diagram the point corresponding to the Natural Haversine of $(p-c)$, the two sides including the required angle, and on the right-hand upright the point for Natural Haversine $(p+c)$.

The line joining the two points may be regarded as a graph which represents the diurnal path of a body of polar distance p, from the Upper to the Lower Transit of meridian. To find the approximate hour angle we have only to take on the graph the point corresponding to the Natural Haversine of zenith distance, and a line drawn through this point parallel to the uprights of the diagram will intersect the base line in the corresponding hour angle.

For the Azimuth the line would be drawn through the points in the uprights corresponding to $(z-c)$ and $(z+c)$, and a point for Nat. Hav. (p) would be marked on the graph. The line through this point parallel to the uprights would intersect the base line in the Azimuth.

The Mnemonic Triangle of the Haversines.

This simple diagram suggests a method of a graphic nature, analogous in some degree to the triangle sometimes employed to impress upon the mind the formulæ connecting

Departure, True and Mer. Diff. Lat., Diff. Long. and Course, which may perhaps be helpful in enabling the "All Haversine" formulæ to be retained in the memory.

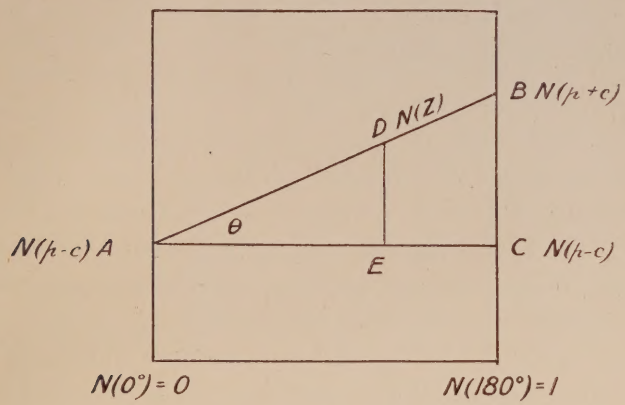

Fig. 9.

In the above diagram, and in the description which follows, the Natural Haversine of an angle is represented by the letter "N."

Thus $N\ (z)$ indicates Natural Haversine of z. Let us suppose that a straight line has been drawn through A, the point in the left hand upright corresponding to $N\ (p-c)$, to B the point in the right hand upright corresponding to $N\ (p+c)$, so that the line $A\ B$ joining the two points is the graph representing the diurnal path. Let D be the point in the line corresponding to $N\ (z)$, where z is a given zenith distance. Then if $D\ E$ is drawn parallel to the uprights of the diagram, $A\ E$ represents $N\ (h)$, the Natural Haversine of the hour angle, which may be measured off on the graduated base line.

Then in the triangle $A\ B\ C$ we have

$$\tan\ \theta = \frac{BC}{AC} = N\ (p+c) - N\ (p-c)$$

since $A\ C$ represents the Natural Haversine of 180° and is therefore equal to unity

Whence in the triangle $A\ D\ E$

$$A\ E = D\ E\ \cot.\ \theta$$

and

$$N\ (h) = \frac{N\ (z) - N\ (p-c)}{N\ (p+c) - N\ (p-c)}$$

Similarly if p, c, h, were given we could with equal ease deduce from the triangle the formula for z, the zenith distance, or for $(p-c)$, which represents the meridian zenith distance in the problem of Reduction to the meridian.

The case of the Altitude Azimuth might be illustrated by a similar diagram, $N\ (z+c)$ and $N\ (z-c)$ being substituted for $N\ (p+c)$, $N\ (p-c)$, $N\ (Z)$ for $N\ (h)$, and $N\ (p)$ for $N\ (z)$.

An Alternative Expression for Zenith Distance.

In view of the increasing popularity of the Marcq method for position lines, in which a single problem, the calculation of zenith distance, takes the place of the older processes, longitude by chronometer, Ex-meridian, etc., having the great recommendation of being available near the meridian and Prime Vertical alike, it becomes a matter of the first importance to provide the navigator with the shortest and simplest method of dealing with this indispensable and constantly recurring operation.

It occurred to the writer some time since that the Haversine formula for zenith distance might be presented in a somewhat novel form, which under the present arrangement of the Table would entail no more labour than the Haversine

and Sine combination which holds the field at present, and which by a certain rearrangement of the Table of Haversines, not less convenient than the present form, would bring about a saving, not indeed in the number of logarithms, etc., employed, but in the necessary number of openings of the Tables, which would be reduced by one. As every practical computer is aware, it is chiefly the process of finding the page which gives trouble, for the right page being found to take out two logarithms instead of one is relatively a small matter.

Before obtaining the new expression it will be necessary to show that

$$\text{Nat. Hav. } \theta + \text{Nat. Hav. } (180 - \theta) = 1$$

Now the Haversine of an angle is equal to the Sine Squared of half the angle.

Thus

$$\text{Hav } \theta = \text{Sin}^2 \frac{\theta}{2}$$

$$\therefore \quad \text{Hav } (180^\circ - \theta) = \text{Sin}^2 (90^\circ - \frac{\theta}{2}) = \text{Cos}^2 \frac{\theta}{2}$$

$$\therefore \quad \text{Hav } \theta + \text{Hav } (180^\circ - \theta) = \text{Sin}^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} = 1.$$

Or expressed in words the Haversine of an angle added to the Haversine of its supplement is equal to unity.

The formula

$$\text{Hav } z = \text{Hav } (p-c) + \left\{ \text{Hav } (p+c) - \text{Hav } (p-c) \right\} \text{Hav } h$$

may be written

$$\text{Hav } z = \text{Hav } (p-c)\,(1 - \text{Hav } h) + \text{Hav } (p+c)\ \text{Hav } h$$

and the formula becomes

$$\text{Hav } z = \text{Hav } (p-c)\ \text{Hav } (12\text{h} - h) + \text{Hav } (p+c)\ \text{Hav } h$$

a formula which it is easy to carry in the memory if it is noted that the two negative signs occur in the same term.

It was communicated privately to the distinguished scientist Captain Alessio, of the Royal Italian Navy, who was sufficiently impressed by its value to make it the subject of a favourable notice in the *Rivista Marittima* of Rome, some extracts from which, with the work of Captain Alessio's own example, I have ventured to translate below:

"But lately the above formula of Goodwin has passed to a form still more worthy of attention—

$$\text{Hav } z = \text{Hav } (p-c) \text{ Hav } (12\text{h}-\text{h}) + \text{Hav } (p+c) \text{ Hav } h.$$

"If, as suggested by Goodwin, the Table of Haversines (Logarithmic and Natural) is arranged to have on the left hand page, the book being open, values of Log Hav and of Nat Hav, for h, $p+c$, $p-c$, etc., and on the right hand page values for the supplements of the various angles, whereby the book of Tables would be affected in no way, either in increased size, or simplicity of arrangement, the formula opens the door to a solution of the problem in no way inferior to the other formulae of the Haversine type.

"Having given for example:

Lat = 20° 34′ N. Dec = 45° 55′ N. h = 2h 14m 58s
c = 69° 26′ p = 44° 5′, 12h − h = 9h 45m 2s.

the calculations will be as follows:

L Hav $(c-p)$ (25° 21′)	8.68256	L Hav $(p+c)$ 113° 31′	9.84479
L Hav (12h − h) (9h 45m 2s)	9.96179	L Hav h (2h 14m 58s)	8.92544
(Sum)	8.64435	(Sum)	8.77023
Nat Hav	.04409		
Nat Hav	.05892		
	.10301		
z =	37° 26′.5		

"As in the Table of Pes which has been mentioned, six openings of the Table will be required, and the numerical operations will be practically equal, but in our opinion the

formula of Goodwin is marked by a symmetry, and, so to speak, an elegance, which, for practical purposes should render it preferable to that of Pes."

Some "Second Difference" Formulae.

In certain problems of Navigation in which, instead of solving a triangle, approximate results are obtained by the use of suitable Tables, the range of such Tables is sometimes greatly extended by the addition of what is known as a "Second Difference" Table, which is generally, though not necessarily, the result of taking into account an additional term in the expansion of some function by that marvellous all-embracing product of mathematical genius, known as Taylor's Theorem. Amongst the problems affected by Second Differences are the clearing of a Lunar Distance, the deduction of G.M.T. from the cleared Distance, the Latitude by Polaris, the Reduction to Prime Vertical, to be described later, and so on.

Application to the Ex-Meridian.

Perhaps in no case is the introduction of an additional correction of this nature of greater importance than in Reduction to the Meridian. The practice of dealing with this problem by means of Tables has progressed greatly during the last half century, but the increase in range of hour angle effected by the addition of a Second Correction Table has not always met with the attention which it merits.

In Vol. I., at page 202, of the 1922 Admiralty Manual will be found an example, worked by the Tables of Inman, which admirably illustrates the value of the Table in question.

The work given in the Manual is as follows:

Data. Lat 30° N., Dec 2° 35′.5 S., Corrected Alt 55° 30′.6 Ship A.T. 0h 49m 22s.

Log H	8.063			
Log *C*	0.206	Alt	55°	30.6′
Log *B*	8.269		2	4.2
B	2° 7.7′	Mer Alt	57	34.8
Corr. Table IV	3.5−	Mer *Z D* (*N*)	32	25.2
		Dec. (*S*)	2	35.5
B (Corrected)	2 4.2			
		Lat	29°	49.7′

Here we find an observation taken more than 21°, or nearly two points of Azimuth, from the meridian, at an hour angle quite outside the limit generally associated with the (Ex-Meridian) problem, unless worked by a logarithmic method, and we find it adjusted to all necessary accuracy by this quantity from Table IV.

For a general description of the Tables made use of the reader may be referred to the Introduction to the Inman Tables, and the formula employed in the calculation of the Table IV., viz.,

$$\frac{x^2}{2} \text{ tan alt sin } 1'$$

where x is the value of the First, or main correction, is stated, but not proved.

Volume I. of the Admiralty Manual in which the example worked above occurs is, we are told in the Preface, "of an elementary and practical nature so that the working only is there shewn;" the explanation of the various processes made use of is therefore reserved for pages 88,

89, of Volume II., for which is claimed "a more advanced character."

The following alternative process to those of the Manual may perhaps be of interest.

Let x represent the First Correction, dz the change of z zenith distance in a time (dh), dh being expressed in minutes of arc.

Then on the kinematic principle described earlier.

$$x = d\ z = \frac{1}{2} \sin Z \cos l\ d\ h,$$

Z being the azimuth at hour angle h,

$$= \frac{1}{2} \frac{\sin h \cos d \cos l\ d\ h}{\sin z}$$

$$= \frac{1}{2} \frac{h \sin 1' \cos d \cos l\ (h)}{\sin z}$$

$$= \frac{1}{2} \frac{\cos d \cos l\ (h^2) \sin 1'}{\sin z}$$

Now in the above expression d, l, h, may be regarded as constants, but z is varying throughout the time h. In the absence of any information as to the amount of x, the "Reduction," it is customary to employ the D.R. meridian zenith distance, given by $(p-c)$, as the z of the equation. But for a correct result we should make use of the half way zenith distance, $z+\frac{x}{2}$.

Our formula would then become, if y indicates the value on this hypothesis

$$y = \frac{1}{2} \frac{\cos d \cos l}{\sin (z+\frac{x}{2})} (d\ h)^2 \sin 1'$$

$$= \frac{1}{2} \frac{\cos d \cos l}{\sin z \cos \frac{x}{2} + \cos z \sin \frac{x}{2}} (d\ h)^2 \sin 1'$$

$$= \frac{1}{2} \frac{\cos d \cos l}{\sin z + \frac{x}{2} \sin 1' \cos z} (d h^2) \sin 1'$$

$$= \frac{1}{2} \frac{\cos d \cos l}{\sin z \left(1 + \frac{x}{2} \sin 1' \cot z\right)} (d h)^2 \sin 1'$$

$$= \frac{1}{2} \frac{\cos d \cos l}{\sin z} (d h)^2 \sin 1' \left(1 - \frac{x}{2} \sin 1' \cot z + \ldots\right)$$

$$\text{or } y = x - \frac{x^2}{2} \sin 1' \cot z$$

since $\frac{1}{2} \frac{\cos d \cos l}{\sin z} (d h)^2 \sin 1' = x$, the approximate value of the correction.

Which gives the equation quoted at page XVII of the "Explanation" of Inman's Tables.

An Alternative Solution.

The expression for this very important Second Difference Correction may be obtained in a more brief, if not quite so elementary a manner, as follows:

Let R represent the approximate value given by

$$\frac{1}{2} \frac{\cos. l \cos d h^2 \sin 1'}{\sin z} = m \operatorname{cosec} z, \text{ suppose}$$

since z is the only variable, and is taken in the first instance at the meridian value, less than it actually is.

Let R^1 represent the value of the expression when z is increased to its proper, or mean value, which is in the first instance unknown.

Then, differentiating R, or m cosec z, with respect to z, we have

$$\frac{d R}{d z} = -m \operatorname{cosec} z \cot z$$

$$\text{But} \qquad R^1 = R + \frac{d R}{d z} (\triangle z) + \ldots$$

and, as in the previous proof, $\triangle z$ should be assumed as one half the change of altitude in the interval, *i.e.*, as $\frac{1}{2} R$.

$$\text{Thus } \frac{d R}{d z} (\triangle z) = - m \operatorname{Cosec} z \operatorname{Cot} z \frac{R}{2}$$

$$= - R \cot z \frac{R}{2}$$

$$\text{and } R^1 = R - \frac{1}{2} \cot z \sin 1' R^2$$

as before.

A Second Difference Table for use with the Ball Altitude Tables (See Brown's Nautical Almanac for 1925, p. 138).

The Ball Altitude Tables are designed to simplify the calculation of zenith distance for laying off a position line in the Marcq system. For each even degree of latitude, hour angle, and declination, the altitude is given to the nearest tenth of a minute, and by selecting suitable values of latitude and longitude from which to work, so as to yield in each case an even degree for latitude and hour angle, all corrections to tabulated altitude are avoided except that required by the odd minutes of declination.

The necessary adjustment for these odd minutes becomes therefore a simple matter of proportion, given by the equation

$$x = \frac{d}{60} A$$

where x is the quantity to be added to, or subtracted from, a recorded value of altitude, A the change of altitude for the degree of declination, and d the odd minutes in the declination, the process being further simplified by a Table giving the values of x at sight for given values of d and A.

It is of course essential to the accuracy of a method of this nature that the change in altitude relative to the change

of declination should be proceeded with a certain degree of uniformity, which is very generally the case. Thus in latitude 24° N. for declinations from 5° N. to 7° N. at an hour angle of 12 minutes, we find the following altitudes:

Dec 5° N	Dec 6° N	Dec 7° N
70° 47.1′	71° 46.3′	72° 45.3′

So that between 5° and 6° of declination we have a change of declination of 59.2′, and between 6° and 7° a change of 59′, shewing that between these values of declination the principle of proportion may be depended upon for a correct result.

If however, retaining our hour angle of 12 minutes, we shift latitude to 3° N., we find the following results:

Altitudes.

Dec 5° N	Dec 6° N	Dec 7° N
86° 24.0′	85° 45.8′	85° 0.4′

Here we find differences of 38.2′ and 45.4′ respectively, indicating that some form of correction in the nature of a Second Difference would be appropriate.

Investigation of an Expression for the Second Difference.

In the ordinary "Triangle of Position" $P\,X\,Z$, with the usual notation, let c=colatitude, p=polar distance, z=zenith distance, and h=hour angle.

Then c and h being considered constant, the problem with which we are concerned is to find the amount by which the zenith distance z exceeds, or falls short of, the value tabulated for the exact degree of declination next lower than the actual value, thus compensating in the zenith distance for the odd minutes in the value of the declination.

Let z be the value in the Table for the next lower degree of declination.

Then

$$z=z, +\frac{d\,z}{d\,p}(\triangle p) + \frac{1}{2}\frac{d^2\,z}{d\,p^2}(\triangle p)^2 - (1)$$

It may easily be shown geometrically that

$$\frac{d\,z}{d\,p}=\text{Cos } P\,X\,Z, \text{ or Cos } X.$$

To find $\frac{d^2\,z}{d\,p^2}$ we have

$$\frac{d^2\,z}{d\,p^2} = -\sin X\,\frac{d\,X}{d\,p}.$$

We have therefore to obtain an expression for $\frac{d\,X}{d\,p}$

In the triangle $P\,X\,Z$

$$\text{Cot } c \sin p=\cot X \sin h+\cos h \cos p.$$

Differentiating

$$\text{Cot } c \cos p = -\sin h \operatorname{cosec}^2 X\,\frac{d\,X}{d\,p} - \cos h \sin p$$

$$\text{Cot } c \cos p + \cos h \sin p = -\sin h \operatorname{cosec}^2 X\,\frac{d\,X}{d\,p}$$

$$\frac{\text{Cos } c \cos p}{\text{sin. } c} + \frac{\cos z - \text{Cos } c \cos p}{\sin c} = -\frac{\sin z}{\sin c \sin X}\frac{d\,X}{d\,p}$$

$$\cos z=-\frac{\sin z}{\sin X}\frac{d\,X}{d\,p}$$

or

$$\frac{d\,X}{d\,p}=-\text{Cot } z \sin X$$

Thus

$$\frac{d^2\,z}{d\,p^2}=-\text{Sin } X\,\frac{d\,X}{d\,p}=\text{Cot } z \sin^2 X.$$

Substituting the values of $\frac{d\,z}{d\,p}$ and $\frac{d^2\,z}{d\,p^2}$ in equation (1) we have

$$z-z_1 =(\triangle p) \cos X + \frac{1}{2}\cot z \sin 1' \sin^2 X\,(\triangle p)^2$$

Since $(\triangle p)$ Cos X is the First Correction, obtained from the Ball Table III., which may be called R.

$$\text{Sin}^2 X\,(\triangle p)^2 = (\triangle p)^2\,(1 - \text{Cos}^2 X)$$
$$= (\triangle p)^2 - R^2.$$

The Second Correction is therefore divided into two parts, the first of which is

$$\frac{1}{2}\,\text{Cot}\,z\,\sin 1'\,(\triangle p)^2$$

where $\triangle p$ represents the odd minutes of declination; while the second is

$$\frac{1}{2}\,\text{Cot}\,z\,\sin 1'\,R^2.$$

A single Table, therefore, may be made to suffice for our purpose, to be entered first with $(\triangle p)$ the odd minutes of declination, and secondly with R, the value of the First Correction; the difference between these quantities, always subtractive, is the value of the Second Correction required.

The following is the Table as given in *Brown's Nautical Almanac*.

(R) First Correction	Altitude					
Odd Minutes of Declination	85°	80°	75°	70°	65°	60°
′	′	′	′	′	′	′
56	5.21	2.59	1.70	1.25	.98	.79
52	4.50	2.23	1,47	1.08	.84	.68
48	3.83	1.90	1.25	.92	.72	.58
44	3.22	1.59	1.05	.78	.60	.49
40	2.66	1.32	.87	.64	.50	.40
36	2.15	1.07	.70	.52	.40	.33
32	1.70	.85	.56	.41	.32	.26
28	1.30	.65	.43	.31	.24	.20
24	.96	.48	.31	.23	.18	.15
20	.66	.33	.22	.16	.12	.10
16	.43	.21	.14	.10	.08	.07
12	.23	.12	.08	.06	.05	.05
8	.12					

And the Rule for its use may be thus stated :

Write down the difference of altitudes taken from the Tables—for the degree of declination containing the value given, and also for the preceding degree, and take the mean of these values.

By means of the Ball Table III., find the first correction (R), employing the mean value obtained as above.

With this value of (R), and again with the odd minutes of declination, enter the Second Correction Table, and take the difference of the quantities obtained.

This difference is the Second Correction, which is always *subtractive*.

Practical Example.

In lat. 3° N., when declination is 6° 40′ N. and hour angle 12m, find the altitude.

From the Tables we have for Lat. 3° N., Hour Angle 12m, altitudes as shewn below.

Dec 5° N	Dec 6° N	Dec 7° N
86° 24.0′	85° 45.8′	85° 0.4′

The differences will be 38.2 and 45.4, giving a mean value of 41.8.

From Ball Table III. for Diff. Dec. 40′, Diff. Alt. 42′, we have the First Correction (R)=28.0′. Entering the special Table under altitude 85° with these values we have for 40′ the value 2.7′, and for 28′ the value of (R) 1.3′, the difference being 1.4′, the Second Correction. The remaining work will be

Altitude for Dec 6°		85° 45.8′
First Correction	-	28.0—
		85 17.8
Second Correction -		1.4—
		85 16.4

which is practically the same as the value obtained by logarithmic calculation.

Reduction to the Prime Vertical.

Something a little less than twenty years ago it occurred to the writer that the fundamental general formula, on which many of the numerous methods for Ex-Meridian are based, need not necessarily be confined to observations for latitude, but might be made to yield equally useful results in finding hour angle from zenith distance and zenith distance from hour angle, the altitude dealt with being taken near the Prime Vertical in the case of the higher latitudes, and of the position of Maximum Azimuth, when the "Angle of Position" is a right angle, nearer to the Equator. Such methods seemed specially adapted to the case of the fixed stars on account of their small variation in declination, and in the year 1906 a work of this kind was published dealing with nearly twenty of the principal stars commonly observed at sea by means of suitable Tables.*

The fundamental formula connecting the zenith distance z of some given body with some previous or subsequent value z of the same body is given by the theorem of Taylor in the following form :

$$z = z^1 + \frac{d\,z}{d\,h}(\triangle h) + \frac{1}{2}\frac{d^2z}{dh^2}(\triangle h)^2 + \frac{1}{6}\frac{d^3z}{d\,h^3}(\triangle h)^3$$

where $\triangle h$ represents a difference in the hour angle of body, p the polar distance, and c the co-latitude, in the typical triangle $P\,X\,Z$, being regarded as constant.

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* *Position Line Star Tables.* Equally adapted to the Marcq St. Hilaire and ordinary Sumner methods. By H. B. Goodwin. London, J. D. Potter, 145 Minories, E. 1. Price Five shillings.

As indicated above, this general formula applies to the case of the Ex-Meridian, which will be first considered.

For $\frac{d\,z}{d\,h}$ we have the well-known expression

Change of zenith distance=Sin Azimuth $\times$Cos Latitude$\times$ change of hour angle.

But when the body is near the meridian, and Sin Azimuth has but an infinitesimal value, the first term vanishes.

We are left therefore with the second term

$$\frac{1}{2}\frac{d^2\,z}{d\,h^2}(\triangle\,h)^2$$

and the various short methods mostly find values from this and tabulate them.

But if z^1 instead of being on the Meridian represents zenith distance on Prime Vertical, then, since Sine Z=Sine 90° =1, we have $\frac{d\,z}{d\,h}$=Cos Lat, which gives a value independent of the declination, and therefore is the same for all heavenly bodies.

On the other hand differentiating $\frac{d\,z}{d\,h}$ to obtain the value of the second term we obtain the value

$$\frac{\text{Cos } l \text{ Cos } d \text{ Cos } P\,Z\,X \text{ Cos } P\,X\,Z}{\text{Sin } z}$$

But if $P\,Z\,X$=90°, Cos $P\,Z\,X$=0, and in this case it is the second term that vanishes.

Pursuing the investigation to another differentiation we find that the third term does not vanish, and finally an expression is arrived at in the following simple form—

$$z-z^1=\text{Cos } l\,(15t)-\frac{1}{6}\text{Cos } l \sin^2 l \sin^2 1'\,(15t)^3+\ldots$$

t minutes of time, or $15t$ in arc, taking the place of $(\triangle\,h)$ in the original formula.

Adaptation to the case of Maximum Azimuth.

In this case, as in the other, the second term vanishes since Cos $P\,X\,Z=0$.

The expression for difference of zenith distance may easily be deduced from the formula established above.

For in this instance, while p and c remain constant, we have the right angle at X.

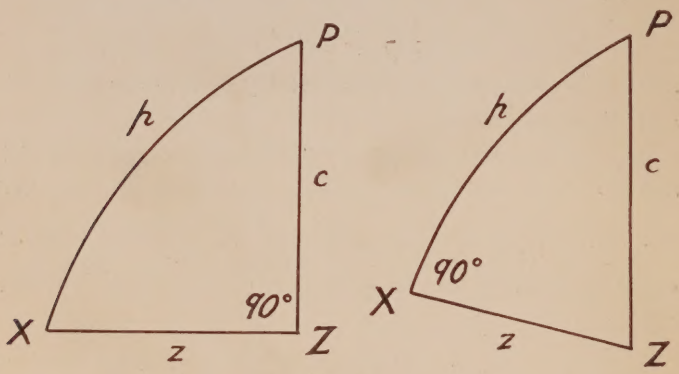

Fig. 10.

From the symmetry of the two triangles, therefore, it follows that

$$z-z^1=\text{Cos } d\,(15t)-\frac{1}{6}\cos d\sin^2 d\sin^2 1'\,(15t)^3$$

The same form of Table will therefore be available, d being employed as argument instead of l.

Construction and Use of Tables.

From what has been said it will easily be seen that a Table may be constructed from the formulæ in the form which follows.

TABLE I.

BETELGUESE (Declination 7° 23′ N.).

Latitude 60° N to 8° 40′ N	Hour Angle	Correction for Change of 1′ in Declination (*a*)	Zenith Distance	Seconds of Time for change of 1° in Z *D* (*b*)	Change in Z *D* per Minute of Time (*c*)
° ′	h m s	s	° ′ ″	s	′
60	5 42 50	6.9	81 28 0	480.0	7.50
59 40	42 36	6.8	26 15	475.2	7.58
20	42 22	6.7	24 30	470.5	7.65
0	42 8	6.6	22 45	466.0	7.73
58 40	41 54	6.5	20 45	461.5	7.80
20	41 40	6.4	19 0	457.2	7.87
0	41 25	6.4	17 0	452.9	7.95

(*a*) Increased declination *add*; decreased declination *subtract*.

(*b*) For use in finding Hour Angle from Zenith Distance.

(*c*) For use in finding Zenith Distance from Hour Angle.

Tables for the Second Correction are added in the following form.

TABLE II. (B).

For use in finding Hour Angle from observed Zenith Distance (Additive to First Correction).

Latitude* or Declination	Distance in H.A. from P.V.					
	35m	40m	45m	50m	55m	60m
°	s	s	s	s	s	s
60	6.2	9.2	13.0	17.8	24.0	31,0
55	5.4	8.1	11.7	15.9	21.3	27.6
50	4.8	7.2	10.3	14.0	18.6	24.1
45	4.1	6.1	8.7	11.9	15.8	20.6
40	3.4	5.1	7.2	9.8	13.1	17.0
35	2.7	4.0	5.7	7.8	10.4	13.5
30	2.1	3.1	4.3	5.9	7.9	10.3

TABLE II. (C).

For use in calculating Z D corresponding to a given Hour Angle (Subtractive from First Correction).

Latitude* or Declination	Distance in Hour Angle from P V or Maximum Azimuth.					
	35m	40m	45m	50m	55m	60m
°	″	′ ″	′ ″	′ ″	′ ″	′ ″
60	46	1 9	1 38	2 14	2 59	3 51
55	47	1 10	1 40	2 17	3 3	3 57
50	46	1 9	1 39	2 15	2 59	3 53
45	43	1 5	1 32	2 6	2 36	3 39
40	39	0 58	1 22	1 53	2 30	3 15
35	33	0 49	1 10	1 36	2 8	2 46

* For observations near Prime Vertical, Latitude of Place is the Argument; near Maximum Azimuth, Declination of Star.

As an illustration for finding Hour Angle, the following example may be given.

On April 11th, 1923, Betelguese had an altitude of 16° 32′ W. of meridian in latitude 60° N. Required the hour angle. Betelguese having only a very trifling annual variation in declination, no correction to tabulated hour angle is required on that account, the declination in 1923 having still approximately the tabular value of 7° 23′ N.

From Table, Zenith Distance on P V	81°	28′	
Observed Zenith Distance - -	73	28	
Diff.	- 8	0	

First Correction = 480s × 8 = 3840s = 1h 4m.

Second Correction, for 64m distance in hour angle from Prime Vertical, latitude 60°, is 38s additive to First Correction.

Hour Angle from Table	5h	42m	50
Sum of Corrections	1	4	38 —
Hour Angle	4	38	12

Logarithmic calculation gives 4h 38m 11s, so that more than an hour from the Prime Vertical we obtain a result practically identical with that obtained by rigorous calculation.

In the following example variation for change of declination has to be taken into account.

April 9th, 1923, in latitude 38° 20′ N. the zenith distance of Regulus W. of meridian was 59° 32′ 15″. Required the hour angle.

The necessary extract from the Tables is given below.

REGULUS (Dec 12° 26′ N.).

Latitude	Hour Angle	Correction for change in Decn.	Z D	Secs. of Time per 1° change of Z D
38° 20	4h 55m 14s	3.0s	69° 41′ 15″	306.0

Declination having changed from the tabulated value of 12° 26′ to 12° 20′ we have first to correct the hour angle of the Table for a difference of 6′ *i.e.*, to subtract 6×3s=18s from the value tabulated for hour angle. The hour angle to be made use of is therefore 4h 54m 56s.

Z D at P V	69°	41′	15″
Z D observed	59	32	15
Diff.	10	9	0 or 10.15°

Then 10.15×306s=3105s.9
=51m 46s.

From Table II. (B) for latitude 38° 20′, hour angle distance 52m, the Second Correction is 11 seconds additive.

Hour Angle on P V	4h	54m	56s.
Sum of Corrections		51	57 —
	4	2	59

Calculation of Zenith Distance from Hour Angle.

The example which follows illustrates the process for the converse operation for zenith distance from hour angle.

In latitude 50° on February 6th, 1923, the hour angle of Aldebaran (Dec. 16° 21′ N) was 6h 7m 37s. Required the zenith distance.

From the Tables we have—

ALDEBARAN (Dec 16° 19′ N.).

Latitude	Hour Angle	Correction for 1′ in Decn.	Z D	Change of Z D per minute
50°	5h 3m7s	4.6s	68° 29′ 0″	9.64′

For 2′ increase in declination we have correction to hour angle $2 \times 4s.6 = 9s$, increasing hour angle to 5h 3m 16s.

Hour Angle star	6h	7m	37s
,, Table	5	3	16
	1	4	21 or 64.35m

Then First Correction $= 9.64' \times 64.35 = 620.3'$ or 10° 20.3′

Second Correction for distance in hour angle 65m, and latitude 50° is −5′ nearly, giving the net difference 10° 15′

Then *Z D* on Prime Vertical	68°	29′
First and Second Corrections	10	15+
Sum	78°	44′

The methods for maximum Azimuth, applicable chiefly in latitudes near the Equator, are similar in principle to those for the Prime Vertical in higher latitudes, so that it is not necessary to give numerical examples in this case.

The various methods, for latitude near meridian and for hour angle and altitude, near Prime Vertical, or position of Maximum Altitude, are fully treated theoretically and practically in the work entitled "Position Line Star Tables," to which reference has been made earlier. The methods as there set forth have elicited expressions of approval of a very gratifying character from the most competent authorities in the United States, Italy, France, and other foreign countries, and in particular the author is glad to take this opportunity of offering his very sincere acknowledgments to Professor Dr. Anthues Stupar for the honour done him by the inclusion of the whole group of formulæ in his *Lehrbuch der Astronomischen Navigation*, published at Fiume in the year 1908; an honour which is the more conspicuous inasmuch as these appear to be almost the only short methods recognised in the work in question.

The Coutinho Short Method for Zenith Distance.

In the spring of the year 1922, two Portuguese naval officers—Rear Admiral Carlos Gago Coutinho, and Commander Sacadura Cabral, accomplished an epoch-making feat in an adventurous voyage by air from Lisbon to Rio. A detailed record of their experiences, carefully maintained throughout, was supplied by these officers to the "Anais do Club Militar Naval" of Lisbon, and cannot fail to be of great service to future navigators, more particularly as the "Club Militar Naval," with commendable public spirit, have re-issued the account in the form of a pamphlet, translated into the English and German languages.

One device adopted by the aerial voyagers appears to have particular interest, firstly on account of its extreme simplicity, and secondly as likely to possess considerable utility in the navigation both of sea and air. The nature of the device will be easily gathered from the extract from pamphlet, which follows:

"In the neighbourhood of the Equator, and when the Sun has a low declination, as was our case in April, and as is easily deduced from the resolution of the already mentioned triangle, the altitude of the Sun, which nears 90° at Noon, is not differing much from the complement of the hour angle. And, as the variation of the addition of hour angle and altitude is slow, at least in the later hours, it becomes easy to compute a Table for that addition. Therefore starting from the hour angle, an altitude is immediately obtained by means of an interpolation at sight and an elementary subtraction. Such was our case when, on the 18th April, we approached St. Paul's Rocks. The Table used on that occasion, and also an example, will give a

clear notion of the practical execution of this process:

Azimuth	280°
Time of transit	1h 57m 15s
P	*Sum of hour angle and altitude*
4h 20m	89° 43′
30	46
40	49
50	52
5 0	54
10	57
20	90 0
30	2
40	5
50	7

These preparations were of course made in advance, and turned to account in practice in the following manner.

P	(4h 54m 45s)	73° 41′	Altitude
Table for	4h 55m	89 53	(Sum)
Computed altitude		16 12	

Those who are conversant with the Ball Altitude Tables will see at a glance how simple the whole operation becomes with the aid of these Tables, the use of which apparently is unknown in Portugal.

As in the following practical example.

Lat. 21° N., Dec. 10° 37′ N., Hour Angle 3h 37m 40s.

Required the altitude.

From the Tables we have—

	Alt.			Hour Angle	
At 3h 32m	37° 59.5′	+11.7′	(Table III.)	=38° 11.2′	+53°=91° 11.2′
36	37 3.6	+11.7	,,	=37 15.3	+54 =91 15.3
40	36 7.6	+11.7	,,	=37 19.3	+55 =91 19.3
At 3h 37m 30s			Sum	91° 17′	(nearly)
			Hour Angle	54 25	
			Altitude	36 52	

This ingenious notion, it will be observed, by balancing the change of hour angle in one direction, against an almost equal alteration of altitude in the other direction, reduces to a mere matter of inspection what in the ordinary course of affairs would be effected by working out a sum in proportion. That is one advantage, and a second, of hardly less importance, follows from the fact that almost the whole of the work required can easily be performed in advance, a matter of no little significance in the navigation of the air.

The Horary Tables of Blackburne and Davis

(*With "Second Correction" Tables greatly extending their use*).

At various periods during the last 130 years, attempts have been made to do as much for the longitude problem as has been so successfully accomplished for the sister problem of the Ex-Meridian, or latitude problem, viz., to enable the mariner to obtain the necessary hour angle by common arithmetic, without logarithmic aid.

The different Tables have this feature in common, that they supply values of hour angle, generally to the nearest second, for each complete degree of latitude, altitude, and declination up to the solar limit of 23°, and leave the adjustment for odd minutes as an element to be dealt with by the computer.

Two such collections of Horary Tables are at present in circulation in this country, one compiled by Captain H. S. Blackburne, for latitudes 0° to 30°, the other by Mr. Percy Davis, formerly Chief Assistant in the *Nautical Almanac* Office, extending from 0° to 50° of latitude. These works

* *The Excelsior Azimuth and Position Finding Tables.* By H. S. Blackburne. Extra Master, etc. James Brown & Son (Glasgow) Ltd., Glasgow. Price Ten shillings and sixpence.

Chronometer Tables or Hour Angles for selected altitudes between latitudes 0° and 50°. By Percy H. L. Davis. J. D. Potter, 145 Minories, E. Price Ten shillings and sixpence.

differ from some of their predecessors by the inclusion of rates of variation for the elements, similar to those with which the *Nautical Almanac* has made us familiar, whereby the adjustment of hour angle for odd minutes in an element is very greatly facilitated.

Attractive as the general notion of dispensing with logarithms must ever be to a large number of navigators, the utility of the Horary Tables has in the past been greatly circumscribed by the drawback that, without some form of Second Correction Table, the ordinary process of correcting the hour angle for odd minutes in an element by simple proportion becomes untrustworthy for the higher values of latitude, or of declination, when the latter is of Contrary Name.

By means of a simple Second Correction Table, occupying a very inconsiderable amount of space, the limits of latitude and declination over which the Tables become available may be very largely extended, so that, up to a limit of, say, 60° of latitude, they could be used for any observation taken on a bearing sufficiently large for hour angle purposes.

A distinct form of Table is rendered necessary for each of the two Tables now in circulation, for the reason that while the Blackburne rates are given for the commencement of each four minute interval, those of Davis are for the middle of the epoch, and represent therefore the *mean* rate of change of hour angle for each odd minute of change in the element.

The Second Correction for the Blackburne Tables.

In connection with the Blackburne form of Table it may be well to explain that, as the Table does not extend beyond 30° of latitude, it is only for extreme declinations of

Contrary Name that the need for a Second Correction is at all urgent.

It should be mentioned also that in correcting an Hour angle for odd minutes of element, we shall start always for the hour angle next *preceding* the epoch of observation, not from that which happens to be nearest. And further, that the change due to increasing or decreasing declination will be the only case considered, but precisely the same process and Table resulting would apply with the other elements concerned. In these days of Sumner methods, however, since we can always work with the nearest exact degree of latitude, the correction for that element at least may be considered almost superfluous.

Let R represent the odd minutes in the value of the declination, a^s the change of hour angle per minute of declination at the beginning of the interval, b^s at the end. Now the rate of variation to be employed is neither that at the beginning or the middle of the interval, but the rate corresponding to a declination midway between the exact degree next lower than that given, and the same degree increased by $\frac{R}{2}$ minutes, where R is the number of odd minutes.

Thus, let us suppose that in latitude 29° N. when declination is 21° 40′ S. it is required to find the hour angle for altitude 26°.

From the Blackburne Tables we have for Lat. 29° N., Alt 26°

Hour Angle			
2h 45m 45.5s	for Dec. 21° S.	Rate	D—5.09s
2 40 32.3	,, 22 S.	,,	5.36

In this case we should require the rate corresponding to declination 21° 20′ S.

Now the change of rate in 60′ is .27, and R the number of odd minutes is 40. For $\frac{R}{2}$ or 20′ we have the change of rate $\frac{20}{60}(.27)$ or .09, so that the rate to be employed is $-5.09\text{s}-.09\text{s}$ or -5.18s., which represents the average rate of change of hour angle per minute of declination, while the value is changing from 21° to 21° 40′.

If then k represents the odd minutes and a, b are the rates at beginning and end of interval, the change of rate is

$$\frac{k}{2}\frac{(b-a)^{\text{s}}}{60} \text{ or } \frac{k}{120}(b-a)^{\text{s}}$$

and the full increase of hour angle is given by

$$k\,a + \frac{k^2}{120}(b-a)^{\text{s}}$$

of which $k\ a$ results from the usual multiplication, and $\frac{k^2}{120}(b-a)^{\text{s}}$ is supplied by inspection of the Second Correction Table. The use of the Table may be illustrated by the following example.

In latitude 30° N., when the declination is 22° 32′ S., and altitude 30°, required the hour angle.

From the Blackburne Tables we have the following:

Dec.	Hour Angle	D (Var. per 1′ of Decn.)
22° S	2h 4m 32.2s	−7.16s.
23° S.	1 57 6.0	−7.78

Required the hour angle corresponding to declination 22° 32′ S.

For the First Correction we have

$$7.16\text{s}\times 32 = 229.1\text{s or } 3\text{m } 49.1\text{s}.$$

For the argument $(b-a)^{\text{s}}$ in the Second Correction Table we take the difference between −7s.16 and −7.78s, that is, −.62s.

In the Table for the correction we find:

k	$(b-a)^s$	
	.6	.7
30′	4.5s	5.3s
35′	6.1	7.1

For the value of .62s for $(b-a)^s$ we may take $(5.0)^s$ as the Second Correction

Then

2h	4m	32.2s	
	3	49.1	—
2	0	43.1	
		5.0	—
2	0	38.1	

The Second Correction being subtractive because the rate of variation is algebraically decreasing.

A Second Correction for the Davis Tables.

With the same notation as before, a, b denoting rates of variation in Seconds of Time per 1′ change of element, and k the number of odd minutes in element, we may proceed as follows.

The rate actually employed when the figure given in Table for mid-interval is made use of is

$$a^s + \frac{1}{2}(b-a)^s$$

giving $k\left\{a^s + \frac{1}{2}(b-a)^s\right\}$ for change of hour angle.

The rate which should be employed is

$$a^s + \frac{k}{2} \times \frac{1}{60}(b-a)^s$$

With a result for change of hour angle—

$$k\left\{a^s + \frac{k}{120}(b-a)^s\right\}$$

The Second Correction is the difference between the two values, viz.:

$$k \left\{ \begin{matrix} 1 \\ 2 \end{matrix} - \frac{k}{120} \right\} (b-a)^s \text{ or } k \frac{60-k}{120} (b-a)^s$$

additive when rate is *decreasing*, and *subtractive* when rate is *increasing*.

The practical example already given would be worked by the Tables of Davis as follows.

Data: Latitude 30° N., Declination 22° 32′ S., Altitude 30°.

The extracts from the Tables in this case will be

Dec.	Hour Angle	D (Var. for 1′ Decn.)
22° S	2h 4m 32s	−7.43s
23 S	1 57 6	−8.12

In the Tables the rates for D are given as −7.4s and −8.1s, the second place of decimals not being given. This being a test example, the more exact values have been supplied.

For the First Correction.

$7.43s \times 32 = 237.8s$ or 3m 57.8s

For the argument of the Second Table we have the difference between −8.12s and −7.43s or −.69s.

Entering the Tables with 32 odd minutes, and .7s for difference of rate, we have 5.1s for the correction, *additive* because rate is *decreasing*.

Then	2h	4m	32s.
		3	57.8−
	2	0	34.2
			5.1+
	2	0	39.3

Worked by logarithms we should have

Lat.	30°	0 N	L Sec	.06247
Dec.	22	32 S	,,	.03449
	52	32		
	60			
	7	28	½ L Hav	3.81367
	112	32	,,	4.91993
				8.83056

Hour Angle 2h 0m 39s

A result which tallies exactly with the hour angle from the Davis process, and within a second of that given by the Tables of Blackburne, for a somewhat unfavourable case in midwinter.

Specimens of the Second Correction Tables required for the Blackburne and Davis collections are appended, as far as a difference of rates of 1.2s.

Blackburne Horary Tables.

a, b are the tabulated rates of change of hour angle in seconds of time, for odd minutes of the element, for the two consecutive whole degrees between which the actual value of the element lies.

Values of Second Correction in seconds of Time.

Values of "k" Odd minutes in element	Difference of rates (a—b) or (b—a)											
	·1s	·2s	·3s	·4s	·5s	·6s	·7s	·8s	·9s	1·0s	1·1s	1·2s
	s	s	s	s	s	s	s	s	s	s	s	s
5′	·02	·04	·06	·08	·1	·1	·2	·2	·2	·2	·2	·3
10	·08	·2	·3	·4	·5	·6	·7	·7	·8	·8	·9	1·1
15	·19	·4	·6	·8	·9	1·1	1·3	1·5	1·7	1·9	2·1	2·3
20	·33	·7	1·0	1·3	1·7	2·0	2·3	2·7	3·0	3·3	3·7	4·0
25	·52	1·0	1·6	2·1	2·6	3·1	3·6	4·2	4·7	5·2	5·7	6·2
30	·75	1·5	2·3	3·0	3·7	4·5	5·3	6·0	6·8	7·5	8·2	9·0
35	1·0	2·0	3·1	4·1	5·1	6·1	7·1	8·1	9·2	10·2	11·2	12·3
40	1·3	2·7	4·0	5·3	6·7	8·0	9·3	10·7	12·2	13·3	14·7	16·0
45	1·7	3·4	5·0	6·7	8·4	10·1	11·8	13·4	15·2	16·9	18·5	20·2
50	2·1	4·2	6·2	8·3	10·4	12·5	14·6	16·6	18·7	20·8	22·9	25·0
55	2·5	5·0	7·6	10·1	12·6	15·1	17·6	20·1	22·7	25·2	27·7	30·2
60	3·0	6·0	9·0	12·0	15·0	18·0	21·0	24·0	27·0	30·1	32·7	36·0

For a rate increasing (algebraically)*add* the Correction.
For a decreasing rate *subtract* the Correction.

Davis Horary Tables.

a, b are the tabulated rates of change of hour angle in seconds of Time, for odd minutes of the element, for the two consecutive whole degrees between which the element lies.

Values of Second Corrrection in seconds of Time.

Values of "k" Odd minutes of element.	Values of (a−b) differences of Rates.										
	·2s	·3s	·4s	·5s	·6s	·7s	·8 s	·9s	1·0s	1·1s	1·2s
	s	s	s	s	s	s	s	s	s	s	s
5′ or 55′	·5	·7	·9	1·1	1·4	1·6	1·8	2·1	2·3	2·5	2·7
10 ,, 50	·9	1·3	1·7	2·1	2·5	2·9	3·3	3·7	4·2	4·6	5·0
15 ,, 45	1·1	1·7	2·3	2·8	3·4	3·9	4·5	5·1	5·6	6·2	6·7
20 ,, 40	1·3	2·0	2·7	3·3	4·0	4·7	5·3	6·0	6·7	7·3	8·0
25 ,, 35	1·5	2·2	2·9	3·6	4·4	5·1	5·8	6·7	7·3	8·0	8·8
30	1·5	2·3	3·0	3·8	4·5	5·2	6·0	6·8	7·5	8·3	9·0

For a rate increasing (algebraically) *subtract* Correction.
For a rate decreasing *add* Correction.

Some Special Features of the Horary Tables.

The Horary Tables may be regarded as offering a complete solution of the spherical triangle for the case of three sides given to find an angle. It may be of interest to shew how an approximate value may be readily obtained for the two remaining angles which complete the six parts of the triangle, by means of the rates of change A^s (for 1′ of altitude), L^s (for 1′ of latitude), D^s (for 1′ of declination), values of which are given in the Tables.

For the Azimuth (*P Z X in the typical triangle*).

The rate $L^s = 4 \cot Z \sec \text{lat.}$

$$A^s = 4 \operatorname{cosec} Z \sec \text{lat.}$$

Dividing each side of the equations one by the other

$$L = A \cos Z$$

whence either by logarithms, or by means of the Plane Traverse Table, the angle Z is easily derived.

For the Angle of Position (*P X Z in the typical triangle*).

Similarly for the angle $P\,X\,Z$.

Here $D^s = 4 \cot X \sec \text{dec}$

$A^s = 4 \operatorname{cosec} Z \sec \text{lat} = 4 \operatorname{cosec} X \sec \text{dec}$

By division

$$D = A \cos X$$

Applications of these Approximate Results.

One very obvious use of the simple methods described for finding approximate values of X and Z is in connection with Great Circle Sailing. Having given the latitudes and longitudes of two places, one of which lies within the tropics, the initial and final courses may be found as in the following example.

It is required to find the initial and final courses between Port Nolloth (Lat. 29° S., Long. 17° E.) to Pernambuco (Lat. 8° S., Long. 35° W.).

From the Blackburne Tables we have for Lat. 29° S., Dec. 8° S., Hour Angle 3h 27m 37s, the values: Altitude 37°, L—1.02s, D+2.38s, A—4.70s.

For the Initial Course.—In the Plane Traverse Table for Distance 470, Diff. Lat. 102, the Course is $77\frac{1}{2}°$. The negative sign of L shews that the azimuth measured from the elevated Pole is greater than 90°. So also, therefore, is the required Course, which is N. $77\frac{1}{2}°$ W.

For the Final Course—For Distance 470, Diff. Lat. 238 gives Course $59\frac{1}{2}°$, and the final Course is N. $59\frac{1}{2}°$ W.

Application to the Equation of Equal Altitudes.

The Equation of Equal Altitudes in its simplest form may be thus defined. It is the number of seconds of time caused by the change of declination in one half elapsed time between the two observations.

Its value in seconds of time is—

$$4 \cot P X Z \sec d \ (d p)'$$

where $d\,p$ is the change of declination in one half the interval. The correction is *additive* when polar distance, measured from the elevated Pole, is increasing, *subtractive* when decreasing; the rule being reversed in the exceptional case occurring only in tropical latitudes when $P X Z$ is greater than 90°.

Reverting to the value for the Equation given, viz.,

$$4 \cot P X Z \sec d \ (dp)'$$

we have as above

$$4 \cot P X Z \sec d = D^s$$

where D^s is the change of hour angle for a change of 1′ in declination.

Thus the expression becomes

$$D^s \times (d\,p)$$

Now the change of declination per hour is tabulated in the *Nautical Almanac*.

Expressing the rate there given as the decimal of a minute, and multiplying by $\frac{h}{2}$, expressed in hours, we have the value of $d\,p$.

If we call the hourly change of declination θ we have for E the equation the value

$$\mathrm{E} = D^s \times \frac{h}{2} \times \theta$$

or expressed in words:

Multiply the hourly change in declination from *Nautical Almanac* expressed in minutes, by one half the hours of elapsed time, and multiply this product by D, the rate for 1′ from Horary Table. The result gives E^s the Equation required.

Practical Example (from Admiralty Manuals of 1914 and 1922).

Latitude 6° 10′ S., Declination 14° 2′ N. (Hourly variation from *Nautical Almanac* 47.3″ or .79′, half interval 3h 28m or 3.47h. Date 28th April.

From the Horary Table for Lat. 6°, Dec. 14° (Contrary Name), Hour Angle 3h 28m, D=1.31^s.

E^s=1.31^s×3.47×.79

Log	1.31	.1173
,,	3.47	.5403
,,	.79	9.8976
(Sum)		10.5552
		+3.6^s

the equation being additive, because in April the Sun in its progress northward is increasing its distance from the South Pole.

A New Development in Horary Tables.

Quite recently the Horary Tables may be said to have passed into a new stage. The Tables hitherto published have owed their origin to the energy and enterprise of private individuals, but in the year 1923 there was issued officially at Washington by the U.S. Hydrographic Department what is by far the most elaborate collection of such Tables that has up to the present time seen the light.

It is styled

H O No 203.

The Sumner Line of Position.

(*Furnished ready to lay down upon the Chart by means of Tables of Simultaneous Hour Angle and Azimuth of Celestial Bodies.*)

In a massive volume containing nearly 900 pages are

given not only hour angles but Azimuths also, for each degree of altitude from 4° or 5° right up to the meridian, for each degree of latitude from 60° N. to 60° S., and of declination to 27° N. or S., thus giving in respect of declination a zone of 8° more than has hitherto been tabulated, and in latitude an increase of 20° in the zone.

A correction is given for odd minutes of declination, corresponding to the "*D*" of Blackburne and Davis, but none for *L* and *A*, the latitude and altitude corrections. So far as *L* is concerned none is actually required by the Sumner methods, since we may work from the nearest whole degree to the D.R. latitude. The correction for odd minutes of altitude is dealt with as follows. In the first instance the hour angle is found for the nearest complete degree of altitude, and a Sumner line, styled "Preliminary Sumner Line," laid off on the chart. At right angles to this line is the line which leads directly to or from the body observed, on which a point is marked corresponding to the odd minutes of altitude. Through this point is drawn a second Sumner line, parallel to the first, which is the "Required Sumner Line." This mode of treatment, it will be noticed, represents a blend of the old Double Chronometer process with a touch of the later Marcq methods added.

Unlike the earlier collections, which cease to supply values when the conditions become unsuitable for longitude observations, the Tables are continued right up to the meridian, and it may perhaps give the old-time navigator something of a shock to find an example worked in the Introduction on an Azimuth only 18° from the meridian. Even, however, if the values given near the meridian can hardly be of much

service for finding reliable hour angles, they may perhaps be turned to profitable account in other directions, for experience often suggests useful functions for a Table very different in character from those which in the first instance were kept in view.

Thus, in the present instance, it happens that in a small Manual, called *L'Annuaire*, published in Paris by the Bureau des Longitudes, the issue for 1925 contains a somewhat novel form of Table, dealing with what is known as "Civil Twilight." This Civil Twilight is the interval between Sunset or Sunrise and the hour angle of the Sun when its depression below horizon is 6°. This epoch is of special interest as marking approximately the hour angle when bright stars of the first and second magnitudes begin to be sufficiently visible for observation in the evening twilight, or cease to be so in the morning twilight.

Now this volume of Tables *H O* No. 203 seems to offer a rather neat and simple method of finding approximately the hour angle when the Sun has a zenith distance of 96°.

Thus let x=the hour angle required at depression 6°

h= ,, ,, for altitude 6° (from Table)

H= ,, ,, of sunset or sunrise.

A sufficiently accurate value for H may be found from *H O* 203 in the following manner.

With the given latitude as latitude, and the given declination as altitude, take from the column headed declination 0, the corresponding Azimuth. This gives the value in arc of H the hour angle required.

The Necessary Formula.

With the usual notation

$$\text{Cos } x \sin p \sin c = \cos 96^\circ - \cos p \cos c$$

$$\cos h \sin p \sin c = \cos 84^\circ - \cos p \cos c$$

$\therefore$ (Cos x + cos h) sin p sin c = -2 cos p cos c

$\therefore$ Cos x + cos h = -2 cot p cot c = 2 cos H.

With due attention to the algebraic signs this equation might be made to yield a value for cos x. But the expression may be largely simplified as follows—

$$1 - \text{vers } x + 1 - \text{vers } h = 2 - 2 \text{ vers } H.$$

$$\therefore \quad \text{vers } x = 2 \text{ vers } H - \text{vers } h$$

$$\text{or hav } x = 2 \text{ hav } H - \text{hav } h.$$

Practical Example.

In latitude 52° N. when declination is 17° N., find the Hour Angle of the end of Civil Twilight.

From the Tables h (hour angle at altitude 6°) 6h 49m 23s.

For Lat. 52° N., ~~Dec.~~ Alt. 17° ~~N.~~, Declination 0°, Azimuth = H = $\bar{1}13.0^\circ$

hav x = 2 hav H − hav h	
hav H (113°)	.69537
	2
	1.39074
hav h (6h 49m 23s)	.60691
(Diff.) hav x	.78383
x	8h 18m 21s.

The value of H as taken from the Tables is of course only approximate, to the nearest decimal of a degree, but may be considered to be quite accurate enough when, as in this case, a result within a minute of time is all that is required. The haversine formula employed, contrasted with the cosine

ERRATUM.

Page 110. Fourteenth line from bottom. For "17° N." read "Alt. 17°."

[*A Companion to the Azimuth Tables*].

ERRATUM.

Page 110. Fourteenth line from bottom. For "Dec. 17° N" read "Alt. 17°."

formula from which it is derived, offers a somewhat striking illustration of the elegant simplicity presented by the haversine, the one function of the angle which knows no algebraic sign other than the positive one, and which increases steadily in value from zero to unity, as the angle itself progresses from 0 to 180°. A period of a century and a quarter has elapsed since Mendoza Rios conferred this great boon upon navigators, and in this country, owing probably to the fostering care of the renowned Professor Inman of the old Naval College at Portsmouth, its merits have been recognised almost from the first, but it is only of late that a knowledge of the conspicuous advantages offered by the Tables of versines and haversines has been extended to maritime countries in general.

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